

Wave-Packet Effects:

a solution for isospin anomalies in vector-meson decay

Kenji Nishiwaki

(केंजी निशिवाकि ← Kendi Nishiwaki ←
니시와키 겐지 ← केंजी निशिवाकि ← 西脇 健二)

Based on works with Kenzo Ishikawa (Hokkaido)

Osamu Jinnouchi (Tokyo Sci) and Kin-ya Oda (Tokyo Woman's Christian)

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SHIV NADAR
INSTITUTION OF EMINENCE DEEMED TO BE
UNIVERSITY
DELHI NCR



Tokyo Woman's Christian University
東京女子大学

My Career

- ✓ UG (2002—2006) @ Kobe University in Japan
- ✓ MAS (2006—2008) @ Kobe University in Japan
- ✓ PHD (2008—2011) @ Kobe University in Japan
(Ph.D thesis adviser: Prof. C.S.Lim)



Kobe



HRI

- ✓ PD (Oct. 2011—Sep. 2014) @ Harish-Chandra Research Institute (India)
- ✓ PD (Oct. 2014—Sep. 12 2018) @ KIAS (South Korea)
- ✓ PD (Sep. 14 2018—Oct. 31 2019) @ Ruđer Bošković Institute (Croatia)
- ✓ Asst. Prof. (Nov. 4 2019—) @ Shiv Nadar University



[Zagreb, Croatia]



[Seoul, South Korea]

My Career

with some of my students



[Zagreb, Croatia]



Kobe

University in Japan
University in Japan
University in Japan
(C.S.Lim)



HRI

Harish-Chandra Research Institute (India)
KIAS (South Korea)
@ Ruđer Bošković Institute (Croatia)
Nadar University



[Seoul, South Korea]

Intro: BSM

- Physics **B**eyond the **S**tandard **M**odel is necessary due to
 - 📌 discussing issues that the SM cannot mention (e.g., dark matter, dark energy, inflation)
 - 📌 addressing (experimental) anomalies that the SM cannot explain (e.g., ~~$\mu\text{on } g-2$~~ , R_D)

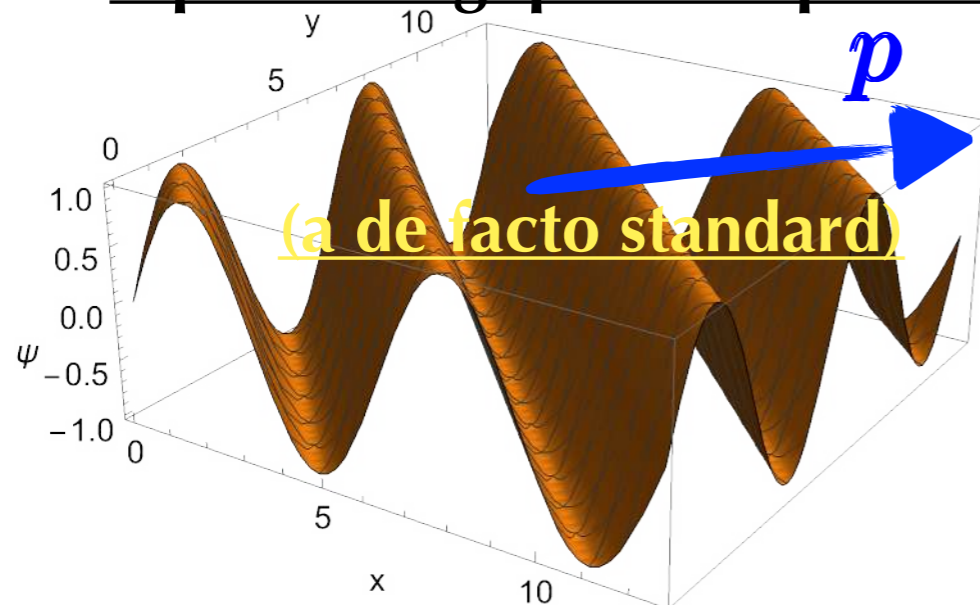
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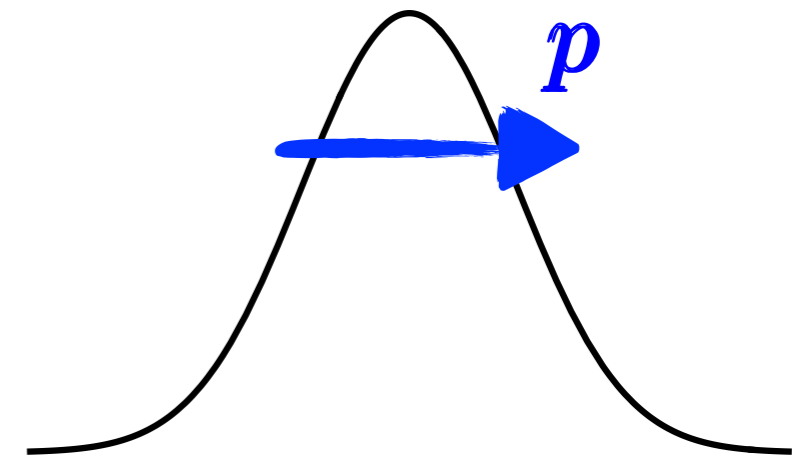
☐ We can focus on another BSM: **B**eyond the **S**tandard **M**ethod:

The plane-wave form is widely used for representing quantum particles.



→
"beyond"

Wave-packet form
for quantum particles

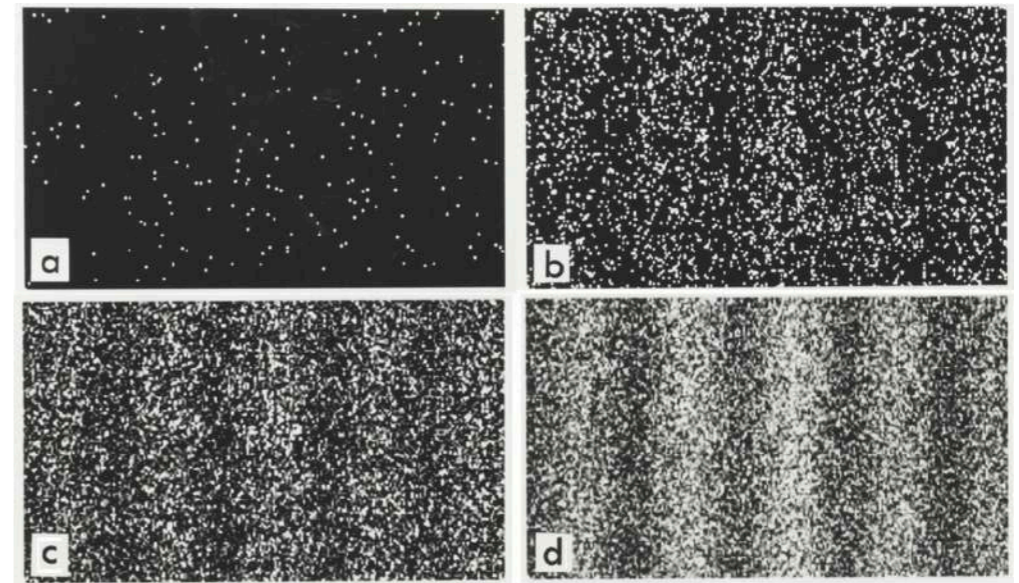
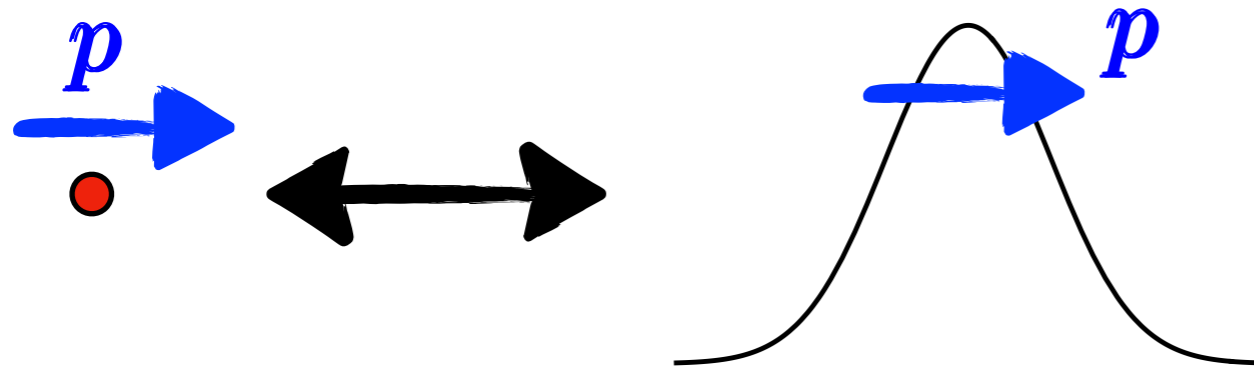


$$\psi(t, \mathbf{x}) \sim e^{-iE_{\mathbf{p}}t + i\mathbf{p}\cdot\mathbf{x}} \quad \left(= e^{+ip_{\mu}x^{\mu}} \right)$$

note: (metric) = diag(-1, 1, 1, 1);
taking afterward: $\hbar = c = 1$

Intro: how about locality?

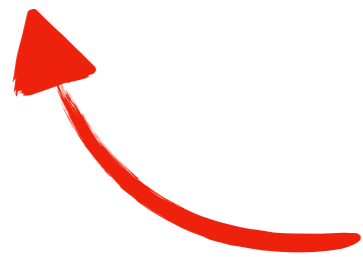
- We remember that wave profiles need to be localised.



[A. Tonomura, Proceedings of the National Academy of Sciences, USA, 102, 14952 (2005)]

- In conclusion,

- The plane-wave description of quantum particles well describes part of necessary properties of particles.
- On the other hand, however, the plane wave **lacks some nature of quantum particles, at least the locality.**



By use of a **localised wave (wave packet)**, we can **overcome** this difficulty and obtain the **full information of quantum transitions!**

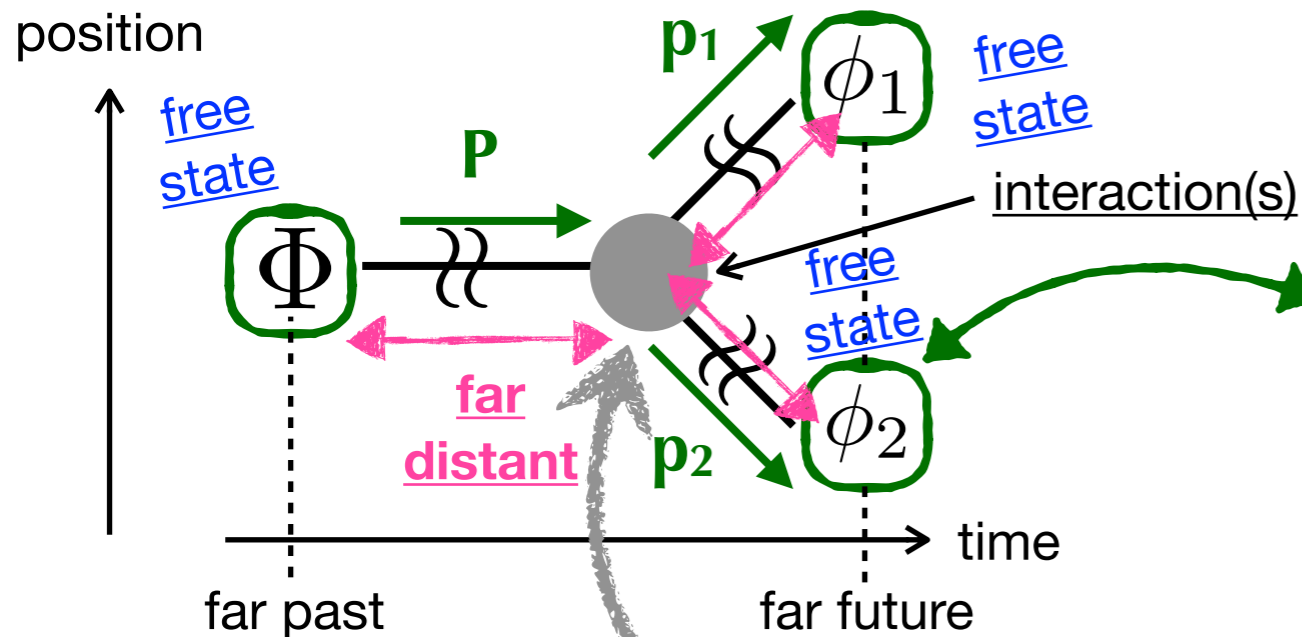
Problem in plane-wave S-matrix

[QFT textbooks]

- ☑ Plane-wave
S-matrix (1 → 2) def.:

$$S_{\text{PW}} = \langle \overset{\text{out}}{\text{free state}} \mathbf{p}_1, \mathbf{p}_2 | \text{T} e^{-i \int_{T_{\text{in}}}^{T_{\text{out}}} dt \hat{H}_{\text{int}}^{(I)}(t)} | \overset{\text{in}}{\text{free state}} \mathbf{P}_0 \rangle$$

$$= \underbrace{(2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}})}_{\text{manifest energy-momentum conservation}} \times \underbrace{(iM_{\text{PW}})}_{\text{(factorised) amplitude}}$$



momentum eigenstates

(external free: also mass eigenstates;
 $E_i^2 = \mathbf{p}_i^2 + m_i^2$)

taking all possible configurations
 (up to an order)

Problem in plane-wave S-matrix

[QFT textbooks]

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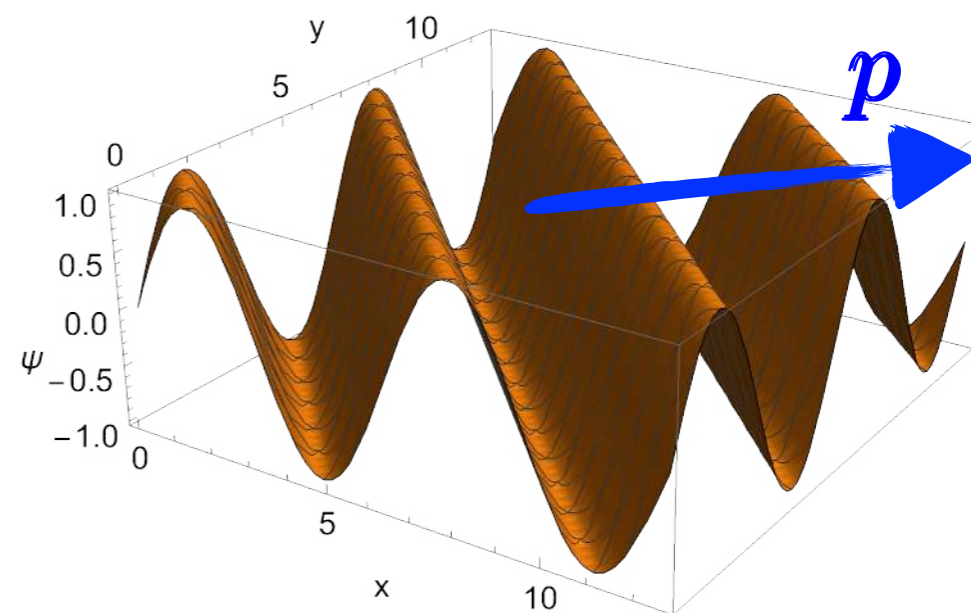
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$$= \underbrace{(2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}})}_{\text{manifest energy-momentum conservation}} \times \underbrace{(iM_{PW})}_{\text{(factorised) amplitude}}$$

- 📌 Corresponding probability is given as $|S_{PW}|^2$. $\underline{(2\pi)^{-4}[(\text{Volume})(\text{Time}) \rightarrow \infty]}$
- 📌 $|S_{PW}|^2$ is ill-defined due to $|\delta^4(P_{\text{out}} - P_{\text{in}})|^2 = \delta^4(P_{\text{out}} - P_{\text{in}}) \times \underline{\delta^4(\mathbf{0})}$.

⇒ **Only the averaged (per V and T) frequencies of events is calculable.**
 ($T_{\text{in}} (= T_{\text{initial}}) = -\infty$, $T_{\text{out}} (= T_{\text{final}}) = +\infty$) \uparrow We will see soon later.

Why the problem happens?
Plane Wave is
non-normalisable!



What is calculable?

□ So, what can we do in the plane-wave formalism?


$$\circ \psi(t, \mathbf{x}) = \frac{1}{\sqrt{2E_{\mathbf{p}}V}} e^{-iE_{\mathbf{p}}t + i\mathbf{p}\cdot\mathbf{x}}$$

“literal normalisation”


$$\circ [(\text{PW}) \text{ phase space}] = \frac{(V)d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{(V)d^3\mathbf{p}_2}{2E_2(2\pi)^3}$$

📌 $|S_{\text{PW}}|^2 \times [\text{phase space}] \times [\text{flux}]$

$$= (2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}}) \frac{1}{2E_{\text{in}}} |M_{\text{PW}}|^2 \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} \times TV$$



well defined
 (The volume is cancelled out.)



ill-defined!
 (since $T, V \rightarrow \infty$)

What is calculable?

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$$\begin{aligned} \bullet |S_{\text{PW}}|^2 \times [\text{phase space}] \times [\text{flux}] \\ = (2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}}) \frac{1}{2E_{\text{in}}} |M_{\text{PW}}|^2 \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} \times TV \end{aligned}$$

$$\frac{|S_{\text{PW}}|^2 \times [\text{phase space}] \times [\text{flux}]}{TV}$$

$$= (2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}}) \frac{1}{2E_{\text{in}}} |M_{\text{PW}}|^2 \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}$$

well defined!

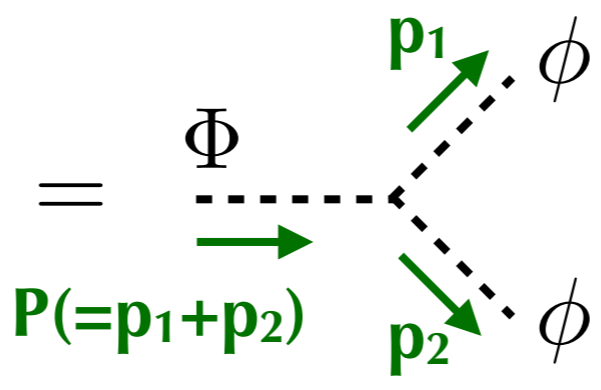
The frequency per time (= Γ : decay rate) is well defined and calculable.

**(SKIPPABLE)
DETAILS**

As we know very well,

□ In the case of $\hat{H}_{\text{int}}(t) = \int d^3\mathbf{x} \frac{\kappa}{2} (\hat{\Phi}\hat{\phi}\hat{\phi}),$

the plane-wave amplitude;
taking a **simple** form,
easily derived via **Feynman rules**

○ $iM_{\text{PW}}(\Phi \rightarrow \phi\phi) =$  $= -i\kappa,$


$\xrightarrow{\text{(for } \mathbf{P}_{\text{in}} = \mathbf{0})}$ $\Gamma(\Phi \rightarrow \phi\phi) = \frac{\kappa^2}{32\pi m_{\Phi}} \sqrt{1 - \frac{4m_{\phi}^2}{m_{\Phi}^2}}$

(SKIPPABLE) ~~Feynman~~ Rules

In ~~theories~~ it is understood that momentum is conserved at each vertex, and that undetermined loop momenta are integrated over: $\int d^4p/(2\pi)^4$. Fermion (including ghost) loops receive an additional factor of (-1) , as explained on page 120. Finally, each diagram can potentially have a symmetry factor, as explained on page 93.

$$\phi^4 \text{ theory: } \mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Scalar propagator:  $= \frac{i}{p^2 - m^2 + i\epsilon}$ (A.1)

ϕ^4 vertex:  $= -i\lambda$ (A.2)

External scalar:  $= 1$ (A.3)

plitude;
orm,
man rules

A.1 Feynman Rules

In **DETAILS** theories it is understood that momentum is conserved at each vertex, and that undetermined loop momenta are integrated over: $\int d^4p/(2\pi)^4$. Fermion (including ghost) loops receive an additional factor of (-1) , as explained on page 120. Finally, each diagram can potentially have a symmetry factor, as explained on page 93.

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$$\text{Scalar propagator: } \begin{array}{c} \longleftarrow \\ p \end{array} = \frac{i}{p^2 - m^2 + i\epsilon} \quad (\text{A.1})$$

The differential decay rate of an unstable particle to a given final state is

$$d\Gamma = \frac{1}{2m_A} \left(\prod_f \frac{d^3p_f}{(2\pi)^3} \frac{1}{2E_f} \right) |\mathcal{M}(m_A \rightarrow \{p_f\})|^2 (2\pi)^4 \delta^{(4)}(p_A - \sum p_f). \quad (\text{A.57})$$

For the special case of a two-particle final state, the Lorentz-invariant phase space takes the simple form

$$\left(\prod_f \int \frac{d^3p_f}{(2\pi)^3} \frac{1}{2E_f} \right) (2\pi)^4 \delta^{(4)}(\sum p_i - \sum p_f) = \int \frac{d\Omega_{\text{cm}}}{4\pi} \frac{1}{8\pi} \left(\frac{2|\mathbf{p}|}{E_{\text{cm}}} \right), \quad (\text{A.58})$$

plitude;
orm,
man rules

S-matrix in Gaussian basis

☑ S-matrix (1 → 2) def.:

[Note: as in the plane-wave basis,
but by the creation/annihilation
operators for wave packets]

$$S := \langle \mathcal{P}_1, \mathcal{P}_2 | \text{T}e^{-i \int_{T_{\text{in}}}^{T_{\text{out}}} dt \hat{H}_{\text{int}}^{(I)}(t)} | \mathcal{P}_0 \rangle$$

$$\left[\mathcal{P}_i = \left\{ \sigma_i, \underbrace{X_i^0 (= T_i), \mathbf{X}_i}_{=: X_i} \right\} \right]$$

This describes the amplitude for the **finite probability/frequency**
of the **event** with **fully-described initial & final particle states!**

“additional”
information

Normalisability of Gaussian
can makes *S* itself finite!

S-matrix in Gaussian basis

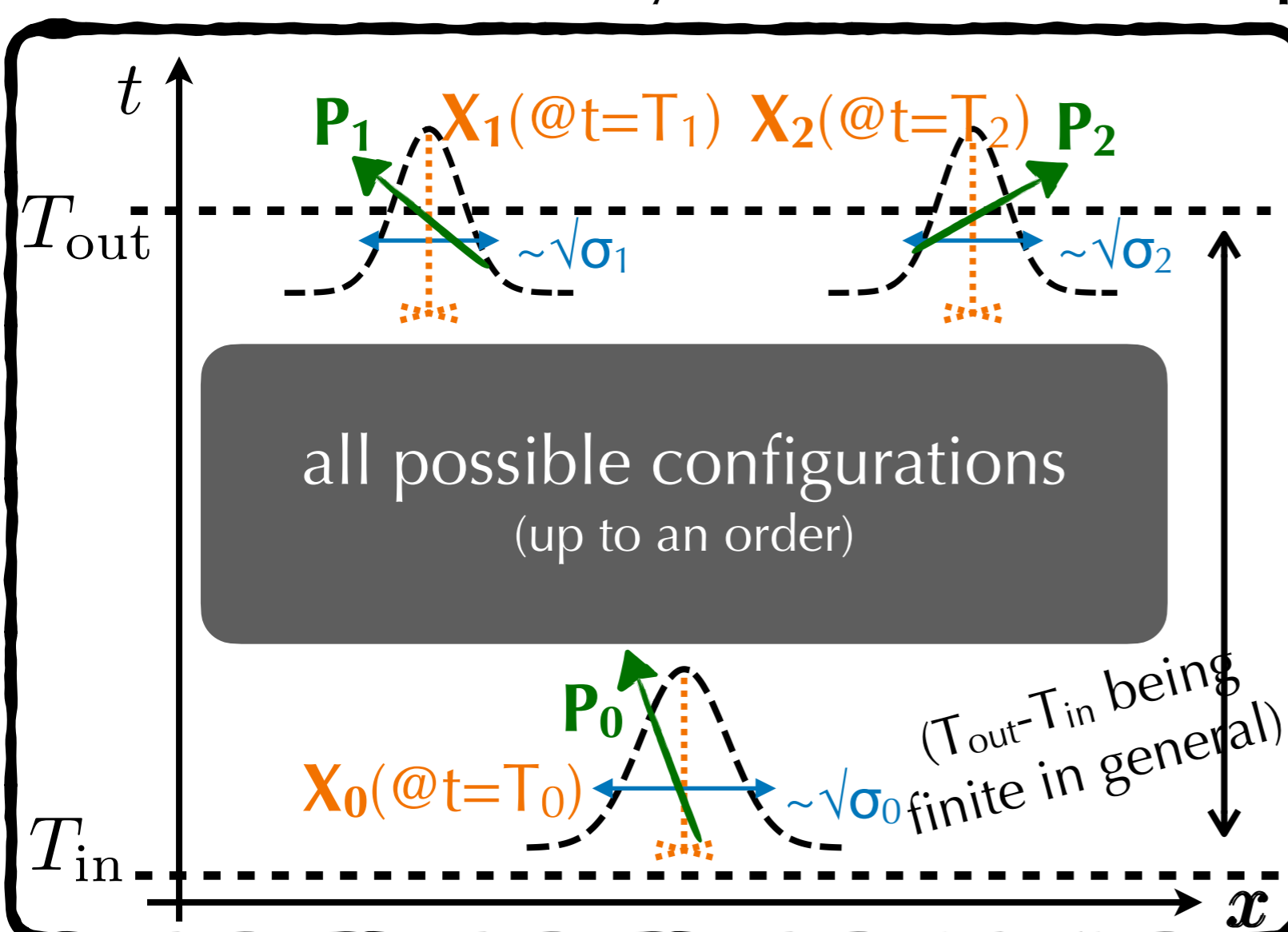
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Normalisability of Gaussian
can makes S itself finite!

○ First proposal by coherent state:
[Ishikawa, Shimomura (0508303)]

○ Claims on various phenomena
by Ishikawa-san et. al.

e.g. [Ishikawa, Jinnouchi, Kubota,
Sloan, Tatsuishi (1901.03019)]

Experiment by the same group → (1907.01264)
($^{22}\text{Na} \rightarrow ^{22}\text{Ne}^* e^+ \nu, e^+(e^-) \rightarrow 2\gamma$)

Short Summary

For the same focused physical $1 \rightarrow 2$ process,

(note: we can similarly construct those for $m \rightarrow n$ processes.)

* plane-wave S-matrix:

- with partial information
- not suitably normalised

$$S_{\text{PW}} = \langle \overset{\text{out}}{\text{free state}} \mathbf{p}_1, \mathbf{p}_2 | \text{T}e^{-i \int_{T_{\text{in}}}^{T_{\text{out}}} dt \hat{H}_{\text{int}}^{(I)}(t)} | \overset{\text{in}}{\text{free state}} \mathbf{P}_0 \rangle$$

$$= (2\pi)^4 \delta^4(P_{\text{out}} - P_{\text{in}}) \times (iM_{\text{PW}})$$

not equal



more informative

suitable limits/marginalisations

* Gaussian S-matrix:

- with full information
- normalised appropriately

$$S := \langle \overset{\text{out}}{\text{free state}} \mathcal{P}_1, \mathcal{P}_2 | \text{T}e^{-i \int_{T_{\text{in}}}^{T_{\text{out}}} dt \hat{H}_{\text{int}}^{(I)}(t)} | \overset{\text{in}}{\text{free state}} \mathcal{P}_0 \rangle$$

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“additional” information

Short Summary

For the same focused physical $1 \rightarrow 2$ process,

(note: we can similarly construct those for $m \rightarrow n$ processes.)

* |plane-wave S-matrix|²:

- with partial information
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$$|S_{\text{PW}}|^2 \quad \text{itself is **ill** defined.} \quad \begin{array}{l} \text{(dimensionful,} \\ \text{relative frequency)} \end{array}$$

$$d\Gamma = \frac{1}{2E_{\text{in}}} \frac{|S_{\text{PW}}|^2}{TV} \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}$$

External states are characterised
by momenta.

* |Gaussian S-matrix|²:

- with **full** information
- **normalised** appropriately

$$|S|^2 \quad \text{itself is **well** defined.} \quad \begin{array}{l} \text{(dimensionless,} \\ \text{absolute frequency)} \end{array}$$

$$dP = |S|^2 \frac{d^3\mathbf{X}_1 d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{X}_2 d^3\mathbf{p}_2}{(2\pi)^3}$$

External states are characterised
by momenta and positions of centres.

Table of Contents

1. Intro: Gaussian S-matrix with “full” information
[6 pages]

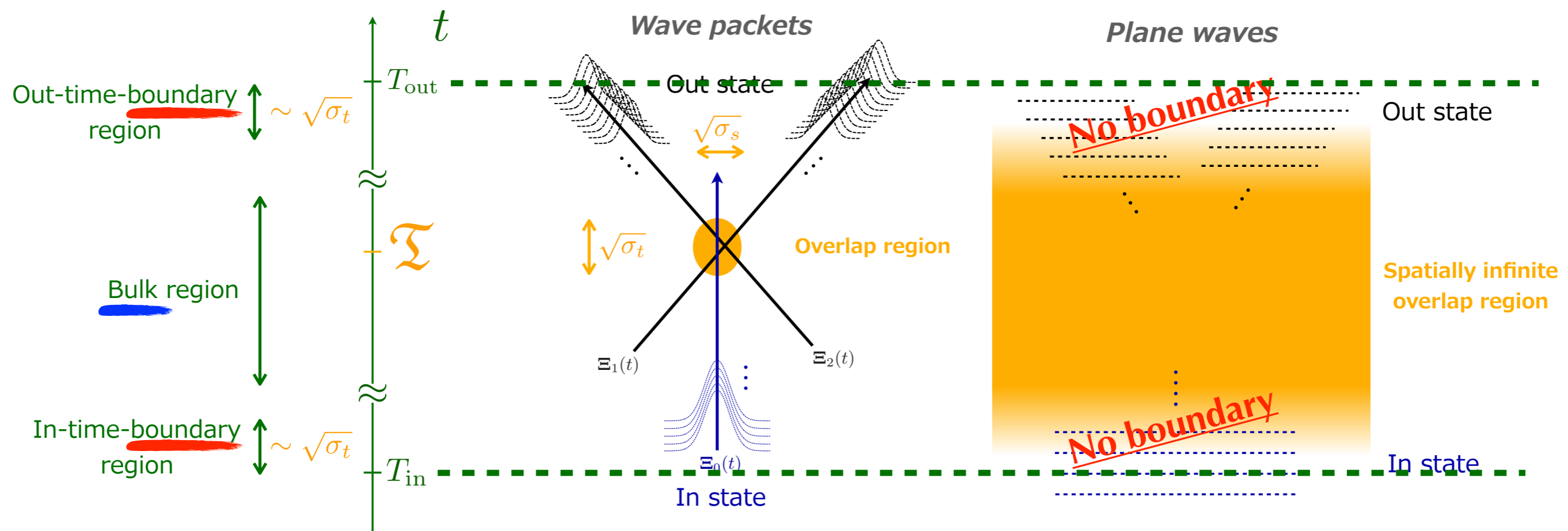
NEXT

**2. Anomalous kinetic effect near mass threshold
(for wave packets) [6 pages]**



**3. Isospin anomalies are resolved via the effect.
[6 pages]**

Structure of Transitions

- ❑ The plane-wave S-matrix has no **time boundaries** (only in **time bulk**).
- ❑ The **wave-packet** S-matrix has **time boundaries** (also in **time bulk**).



What's next?

- We examined the simplest $1 \rightarrow 2$ case in wave-packet formalism.
Now, we will be interested in
 1. How about the $2 \rightarrow 2$ full scattering, including the production part?
 No detailed discussion today
 2. When does the wave-packet effect become significant?
 Today's main topic

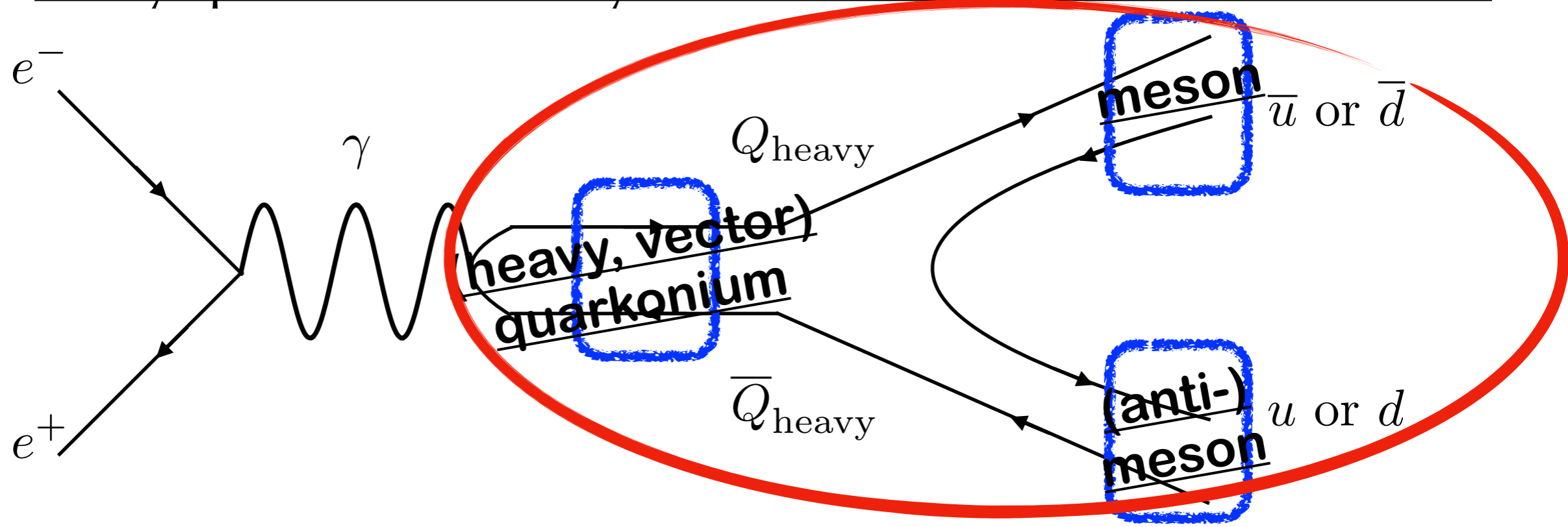
□ So, the `best' process to see a wave-packet intrinsic nature requires

- domination of the boundary contribution, e.g., via a narrow phase space
 - resonant production & decay
- +
- experimental anomalies being reported



We found such a process!

⇒ heavy quarkonium decays into mesons near kinetic threshold



Anomaly in heavy vector quarkonium decays ^{Sec. 2 4/6}

□ For each heavy vector quarkonium (V), two dominant decay branches are “ $V \rightarrow P^+ P^-$ ” and “ $V \rightarrow P^0 \bar{P}^0$ ”.

○ P^+ is the EM-charged one; (anti-particle of P^+) = P^-

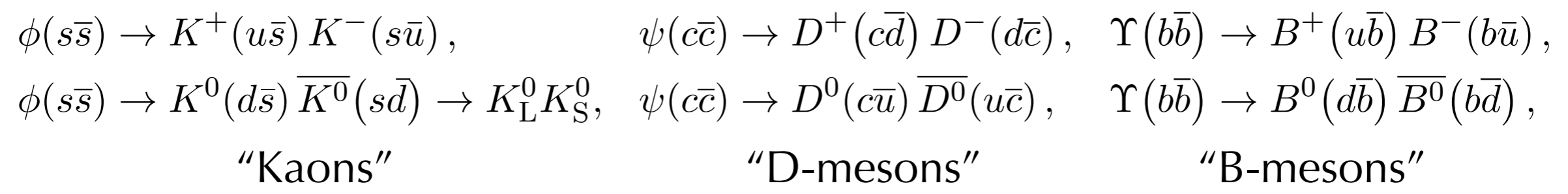
○ P^0 is the EM-neutral one; (anti-particle of P^0) = \bar{P}^0

$$\begin{array}{lll} \phi(s\bar{s}) \rightarrow K^+(u\bar{s}) K^-(s\bar{u}), & \psi(c\bar{c}) \rightarrow D^+(c\bar{d}) D^-(d\bar{c}), & \Upsilon(b\bar{b}) \rightarrow B^+(u\bar{b}) B^-(b\bar{u}), \\ \phi(s\bar{s}) \rightarrow K^0(d\bar{s}) \bar{K}^0(s\bar{d}) \rightarrow K_L^0 K_S^0, & \psi(c\bar{c}) \rightarrow D^0(c\bar{u}) \bar{D}^0(u\bar{c}), & \Upsilon(b\bar{b}) \rightarrow B^0(d\bar{b}) \bar{B}^0(b\bar{d}), \\ \text{“Kaons”} & \text{“D-mesons”} & \text{“B-mesons”} \end{array}$$

Anomaly in heavy vector quarkonium decays

□ For each heavy vector quarkonium (V), two dominant decay branches are “V → P⁺P⁻” and “V → P⁰ $\overline{P^0}$ ”.

- P⁺ is the EM-charged one; (anti-particle of P⁺) = P⁻
- P⁻ is the EM-neutral one; (anti-particle of P⁰) = P⁰



Measuring Isospin breaking

$$R_\phi := \frac{\Gamma(\phi \rightarrow K^+ K^-)}{\Gamma(\phi \rightarrow K_L^0 K_S^0)}, \quad R_\psi := \frac{\Gamma(\psi \rightarrow D^+ D^-)}{\Gamma(\psi \rightarrow D^0 \overline{D^0})}, \quad R_\Upsilon := \frac{\Gamma(\Upsilon \rightarrow B^+ B^-)}{\Gamma(\Upsilon \rightarrow B^0 \overline{B^0})}.$$

○ PDG values:

$$R_\phi^{\text{PDG}} = 1.45 \pm 0.03, \quad R_\psi^{\text{PDG}} = 0.798 \pm 0.010, \quad R_\Upsilon^{\text{PDG}} = 1.058 \pm 0.024.$$

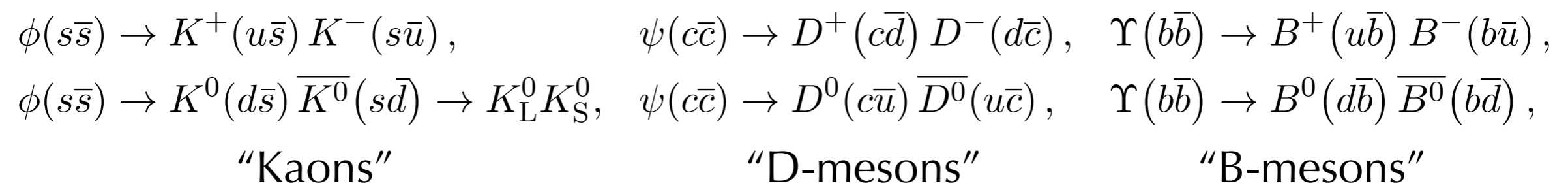
○ plane-wave results:

$$R_\phi^{\text{plane}} = \frac{g_{\phi+}^2}{g_{\phi0}^2} (1.5156 \pm 0.0033), \quad R_\psi^{\text{plane}} = \frac{g_{\psi+}^2}{g_{\psi0}^2} (0.6915 \pm 0.0046), \quad R_\Upsilon^{\text{plane}} = \frac{g_{\Upsilon+}^2}{g_{\Upsilon0}^2} (1.047 \pm 0.026).$$

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↕ 2.1σ

↕ 9.5σ!!

↕ 0.32σ

[for isospin-symmetric case ($g_{P^+} = g_{P^0}$)]

- The mass difference in the final states deviates the ratio from unity.
- In the **plane-wave** calculation, $R(\Phi)$ and $R(\psi)$ depend on only the masses in the isospin-symmetric limit ($g_+ = g_0$).

$$\widehat{\mathcal{H}}_{\text{int,eff}}^{(I)} = ig_{V^+} \mathcal{V}^\mu [\mathcal{P}^+ \partial_\mu \mathcal{P}^- - \mathcal{P}^- \partial_\mu \mathcal{P}^+] + ig_{V^0} \mathcal{V}^\mu [\mathcal{P}^0 \partial_\mu \overline{\mathcal{P}}^0 - \overline{\mathcal{P}}^0 \partial_\mu \mathcal{P}^0]$$

It should be good since $m_u \sim m_d \sim O(1)$ MeV, while $m_s \sim O(10^2)$ MeV and $m_c \sim O(1)$ GeV.

[Branon, Escribano, Lucio, Pancheri, hep-ph/0003273]

$$\begin{aligned} \circ \Gamma(\phi \rightarrow K^+ K^-) &= \frac{2}{3} \left(\frac{g_+^2}{4\pi} \right) \frac{|\mathbf{k}|^3}{m_\phi^2}, \\ |\mathbf{k}| &= \frac{1}{2} (m_\phi^2 - 4m_{K^+}^2)^{1/2} \\ \circ R_{\text{th}} &:= \frac{\Gamma(\phi \rightarrow K^+ K^-)}{\Gamma(\phi \rightarrow K^0 \overline{K}^0)} \Big|_{\text{th}} =: \left(\frac{g_+^2}{g_0^2} \right) R_{\text{FGR2}} \\ &= \cancel{\left(\frac{g_+^2}{g_0^2} \right)} \left(\frac{m_\phi^2 - 4m_{K^+}^2}{m_\phi^2 - 4m_{K^0}^2} \right)^{3/2}. \end{aligned}$$

📌 Isospin breaking and QED corrections do not resolve the discrepancy of $R(\Phi)$.

📌 Here, the form factor $[\psi(r=0)]$ of the vector meson is cancelled out in R .

Note:

$$\begin{aligned} m_\phi &= (1019.461 \pm 0.016) \text{ MeV}, \\ 2m_{K^+} &= (987.354 \pm 0.032) \text{ MeV}, \\ 2m_{K^0} &= (995.222 \pm 0.026) \text{ MeV}, \\ \Gamma_\phi &= (4.249 \pm 0.013) \text{ MeV}, \end{aligned}$$

$$\begin{aligned} m_\psi &= (3773.7 \pm 0.4) \text{ MeV}, \\ 2m_{D^+} &= (3739.32 \pm 0.10) \text{ MeV}, \\ 2m_{D^0} &= (3729.68 \pm 0.10) \text{ MeV}, \\ \Gamma_\psi &= (27.2 \pm 1.0) \text{ MeV}, \end{aligned}$$

$$\begin{aligned} m_\Upsilon &= (10579.4 \pm 1.2) \text{ MeV}, \\ 2m_{B^+} &= (10558.7 \pm 0.24) \text{ MeV}, \\ 2m_{B^0} &= (10559.3 \pm 0.24) \text{ MeV}, \\ \Gamma_\Upsilon &= (20.5 \pm 2.5) \text{ MeV}. \end{aligned}$$

Form of Gaussian wave-packet S-matrix

$$\begin{aligned}
 \checkmark S_{V \rightarrow P\bar{P}} &= ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi\sigma_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta\mathbf{P})^2 - \frac{\mathcal{R}}{2}} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathfrak{T}) \\
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a finite form

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form factor for the vector quarkonium

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Fourier transform & normalising
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[approximate form of (s-wave) ground state under a Coulomb potential (beyond "r=0" approximation)]

In this order, the form factor depends on final-state configurations

→ NOT factored out in R_V.

[Fischbach, Overhauser, Woodahl, hep-ph/0112170]
(c.f., a similar introduction for $\Phi \rightarrow 2K$)

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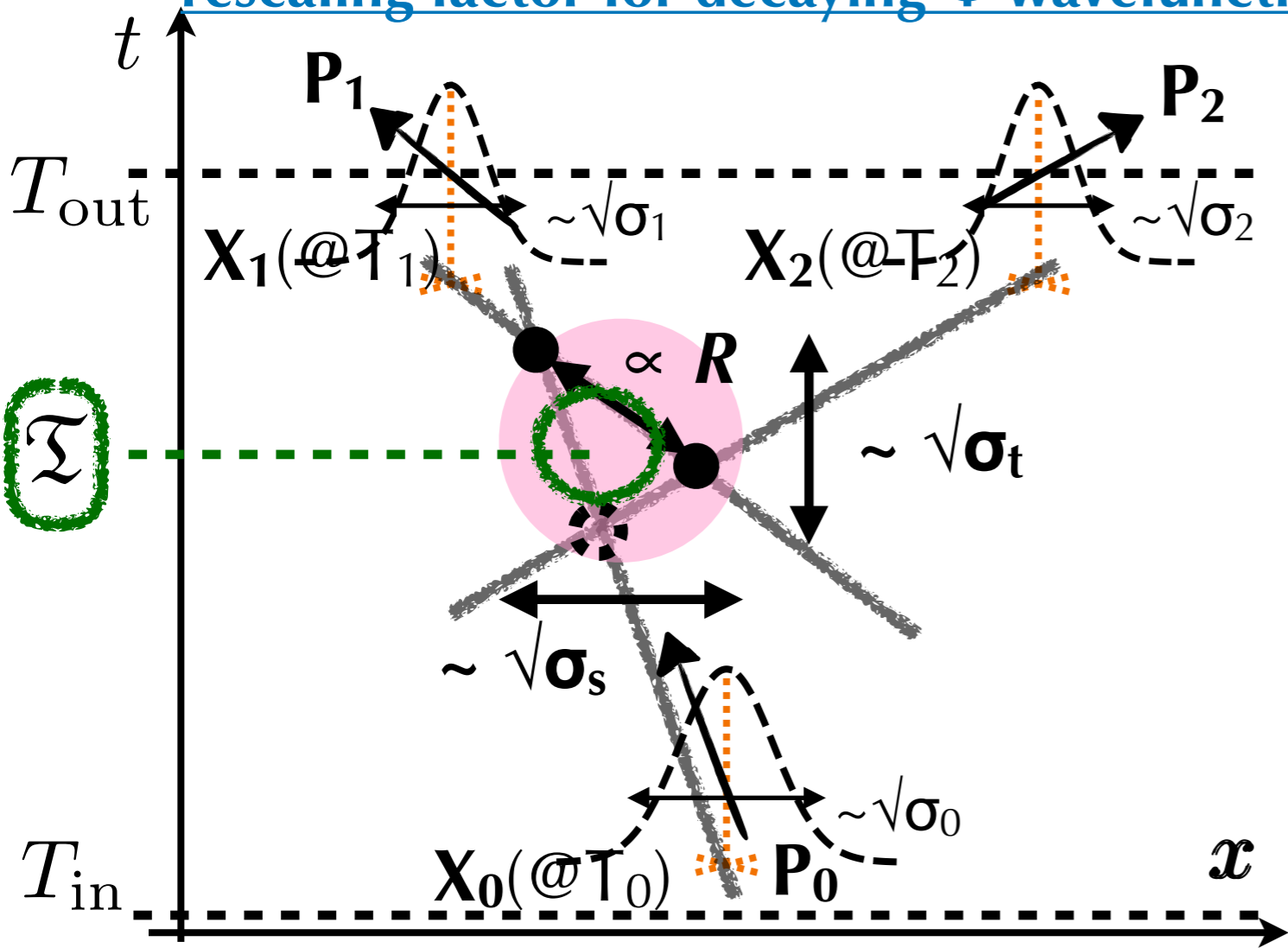
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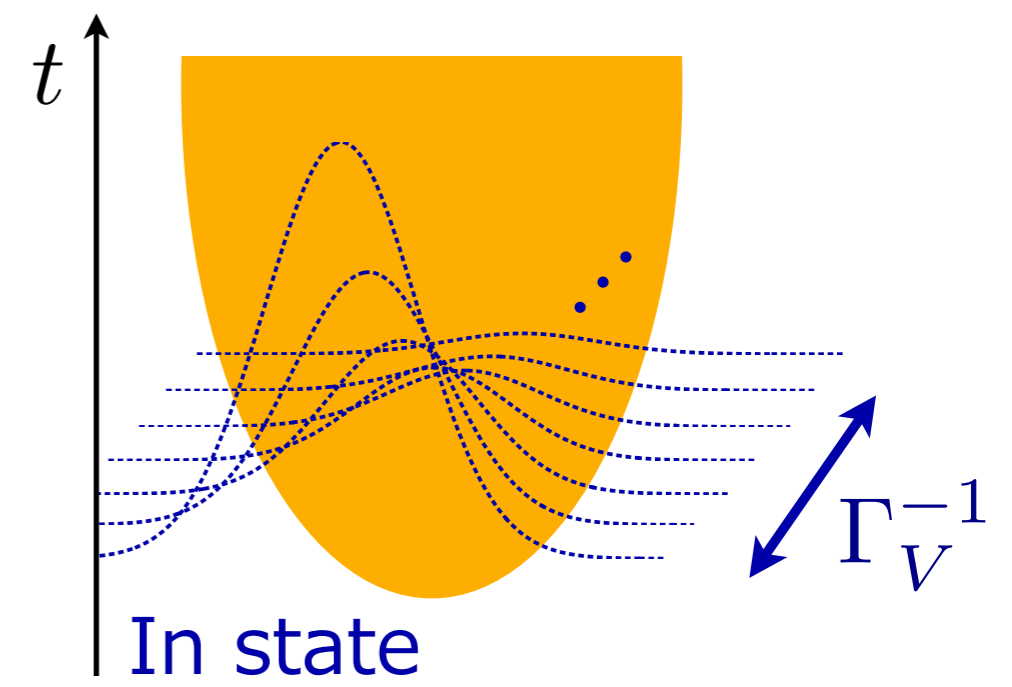
\mathcal{T} : time of overlap (around which three wave packets overlap).

“window function”
 \Rightarrow other “T” dependence

rescaling factor for decaying- Φ wavefunction



The off-shell-ness (decaying nature) of the initial state is taken into account (Weisskopf-Wigner Approx.).
 \Rightarrow Debate on the resonant peak!



In state

Table of Contents

1. Intro: Gaussian S-matrix with “full” information
[6 pages]
2. Anomalous kinetic effect near mass threshold
(for wave packets) [6 pages]
- 3. Isospin anomalies are resolved via the effect.
[6 pages]**

NEXT

Predictions for the Ratio

□ For $\psi \rightarrow D^+ D^-$ and $\psi \rightarrow D^0 \bar{D}^0$

Overall normalisation does not contribute to the ratio $R(\psi)$.

Parameters: $m_\psi, m_{D^+} (= m_{D^-}), m_{D^0} (= m_{\bar{D}^0}), \tau_\psi (= \Gamma_\psi^{-1});$

$R_0, \sigma_D, \sigma_\psi; N_d$
 with saddle-point approx.

Experimental data are available.

wave-packet profile

Parameter of Form factor [Length]

Size of Wave Packet (final D & Dbar) [Length²]

Size of Wave Packet (initial ψ) [Length²]

Note 1:

$$\left[\lambda_{\text{de-Broglie}} = \mathcal{O} (10^{-2} \text{ MeV}^{-1}) \right]$$

$$\lambda_{\text{de-Broglie}} \lesssim \sqrt{\sigma_D}$$

Note 2:

T_{in} provides overall effects (NOT to the ratio).

Renormalisation of initial- ψ 's wavefunction (due to decaying nature)

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[Reminder]

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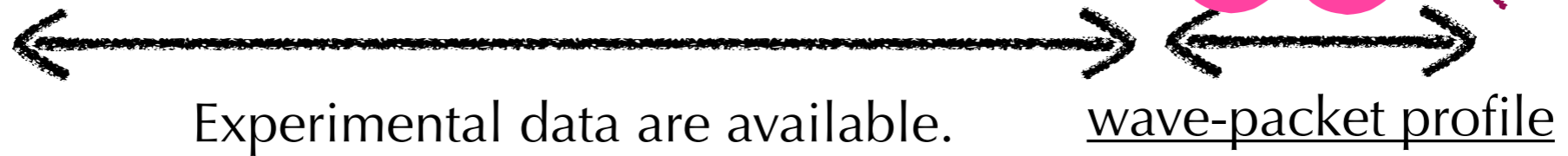
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normalisation
initial- ψ 's
transition
function
decaying
(re)

Predictions for the Ratio

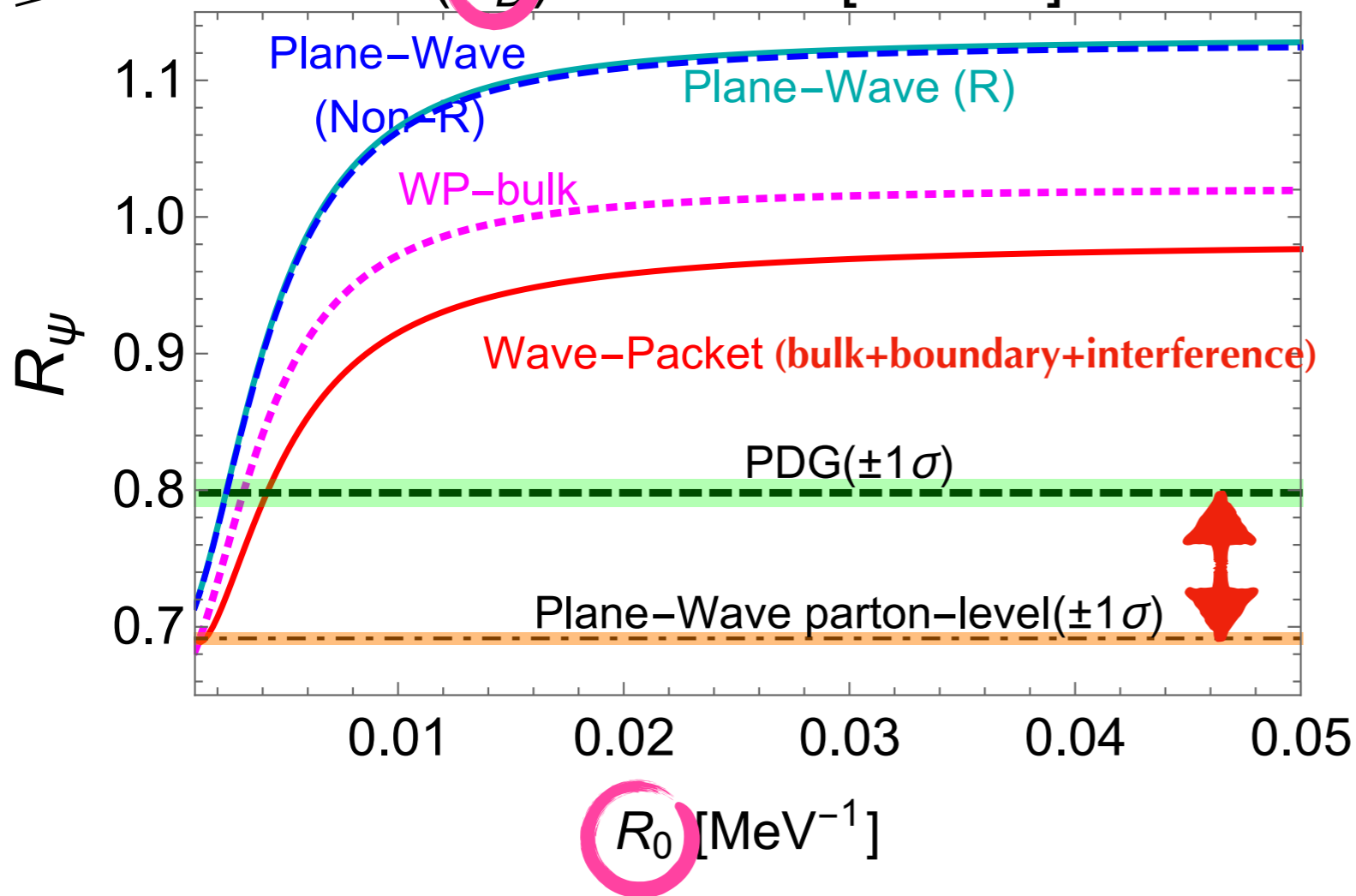
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in a **small** wave packet

$(\sigma_D)^{1/2} = 0.01 \text{ [MeV}^{-1}\text{]}$

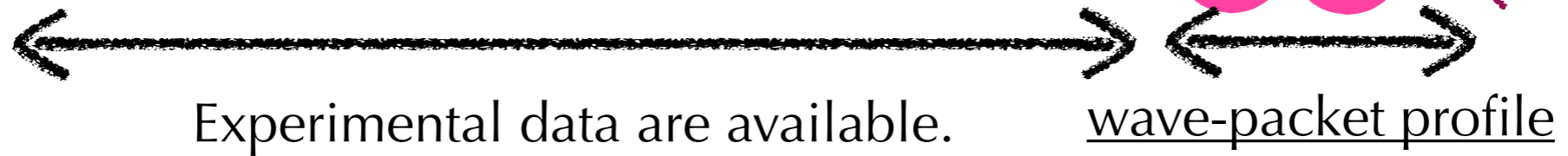


○ **Parton-level (factored form factor) deviated from PDG (as reported).**

Predictions for the Ratio

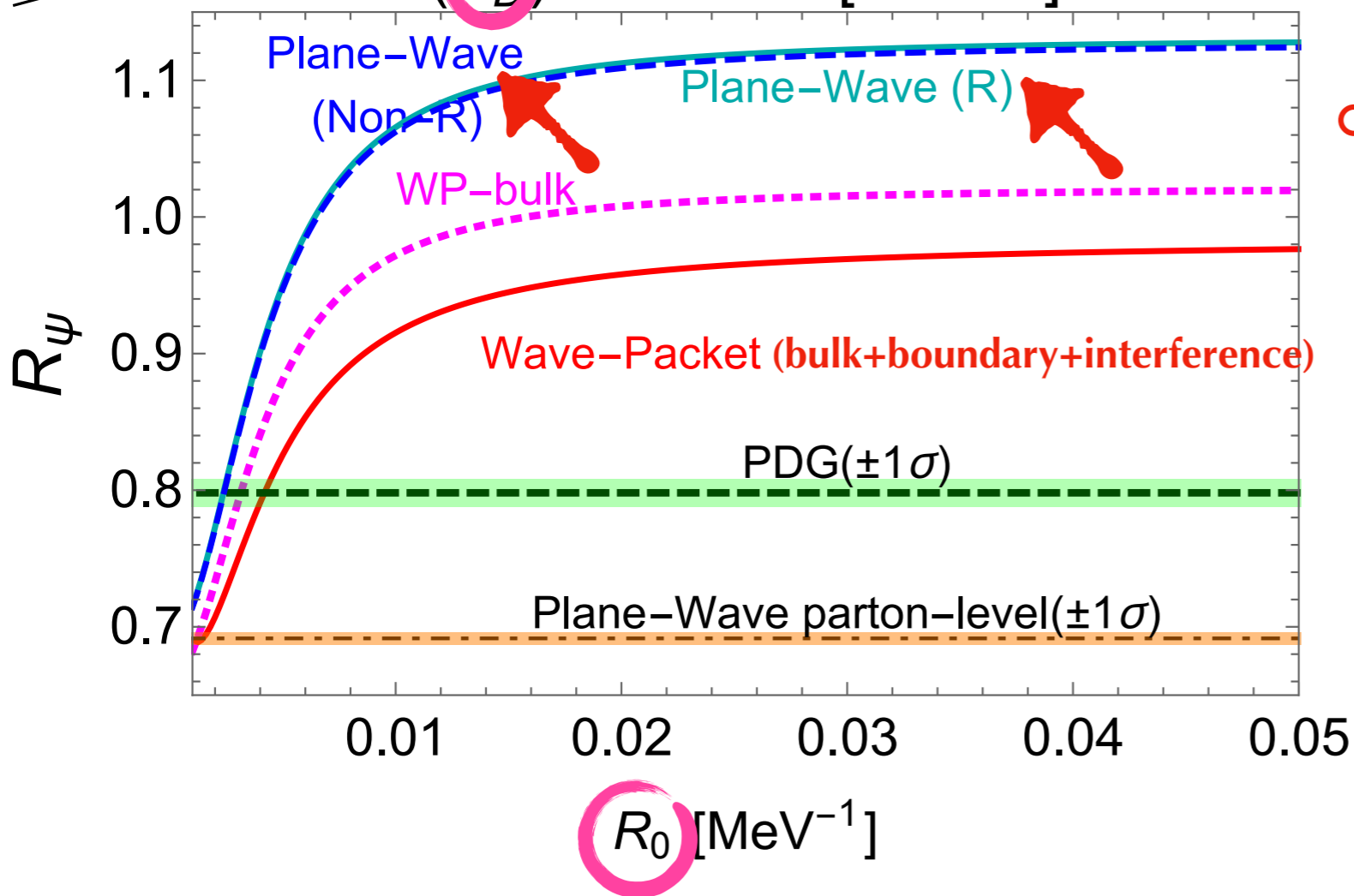
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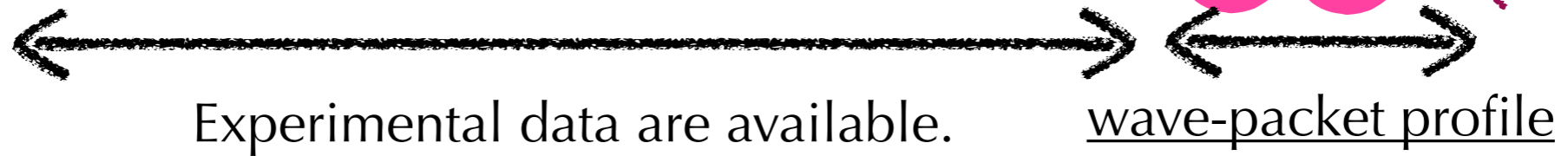
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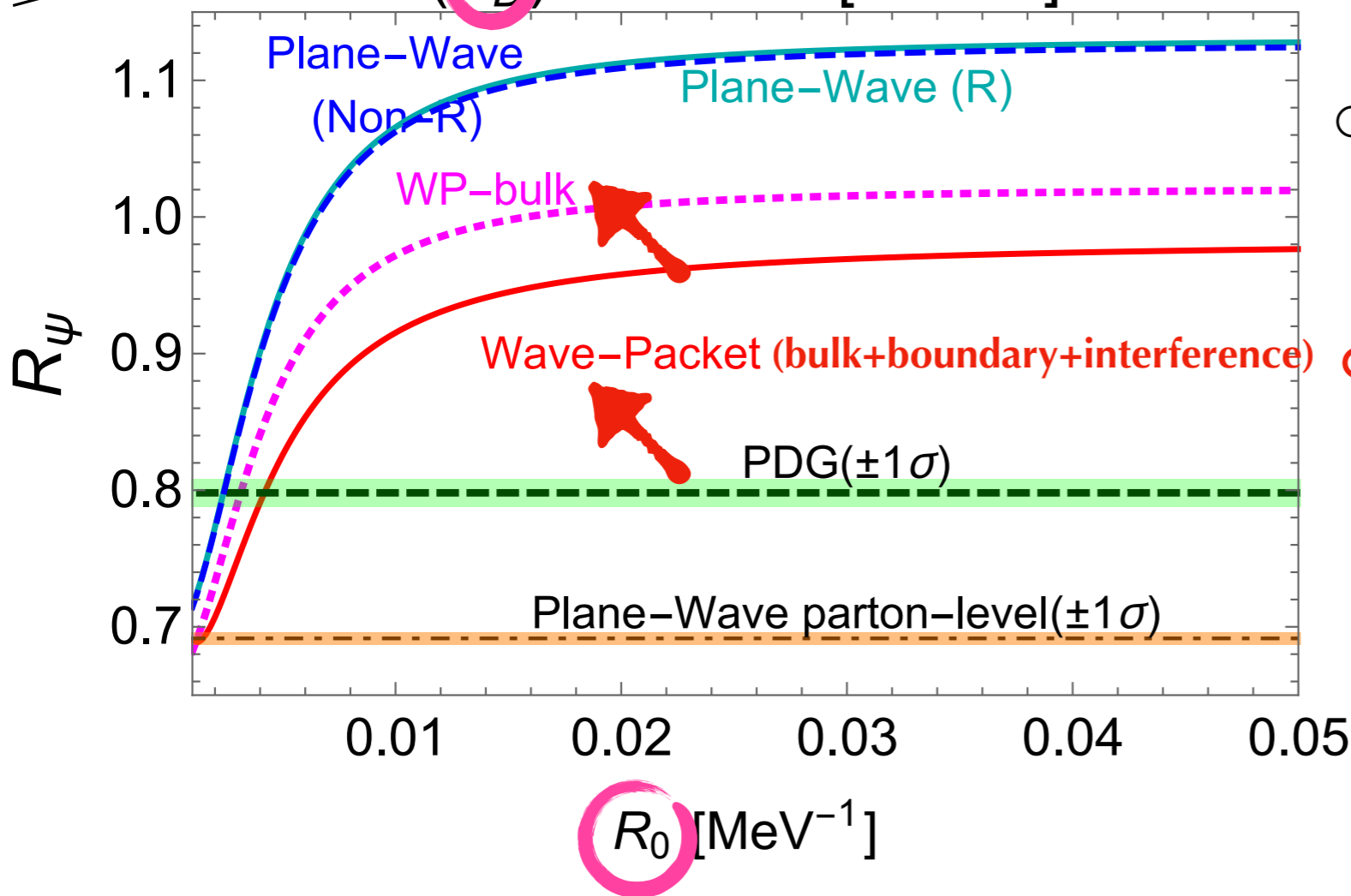
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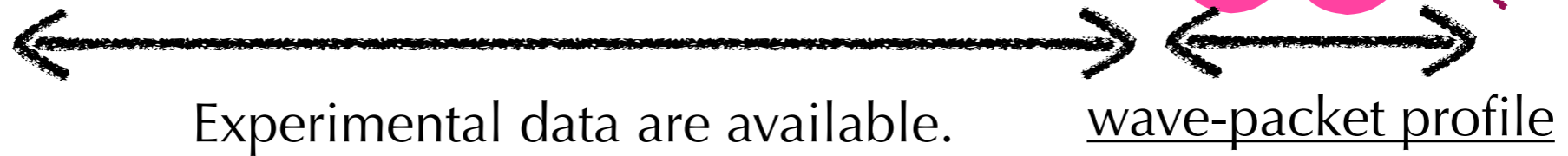


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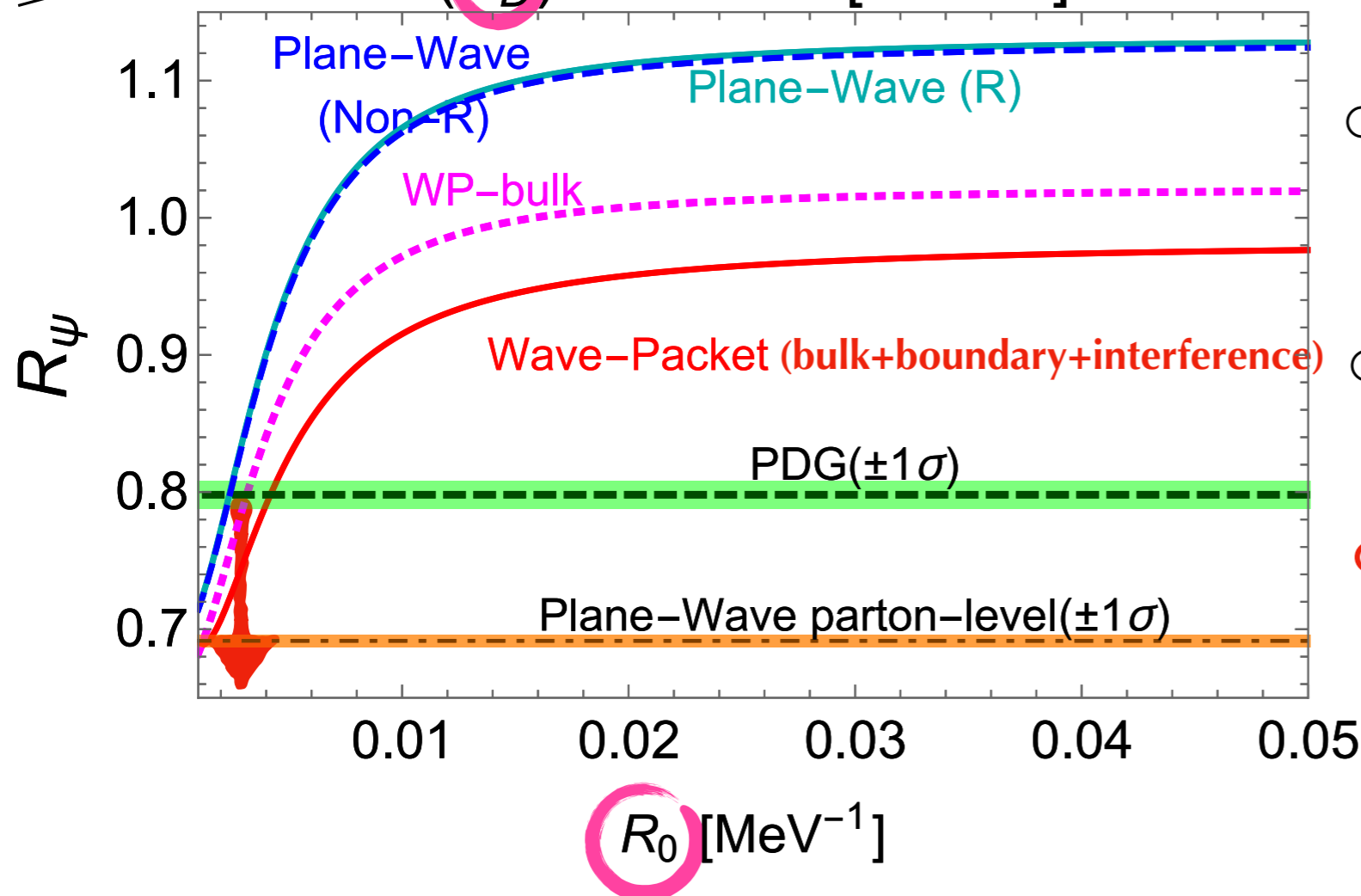
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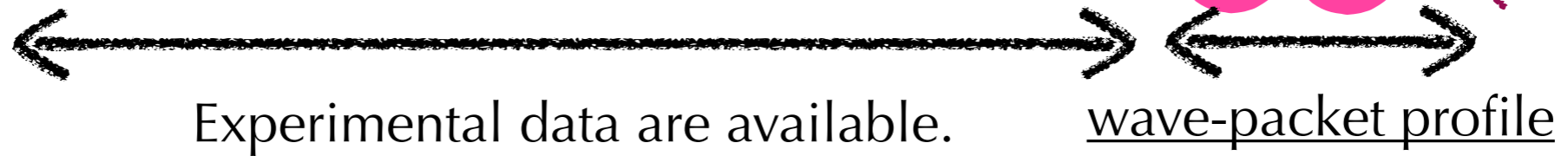
○ Wave-packet calculations explain the PDG result.

○ $R_0 \sim O(10^{-2}) \text{ MeV}^{-1} \sim (\text{QCD scale})^{-1}$ is a reasonable choice.

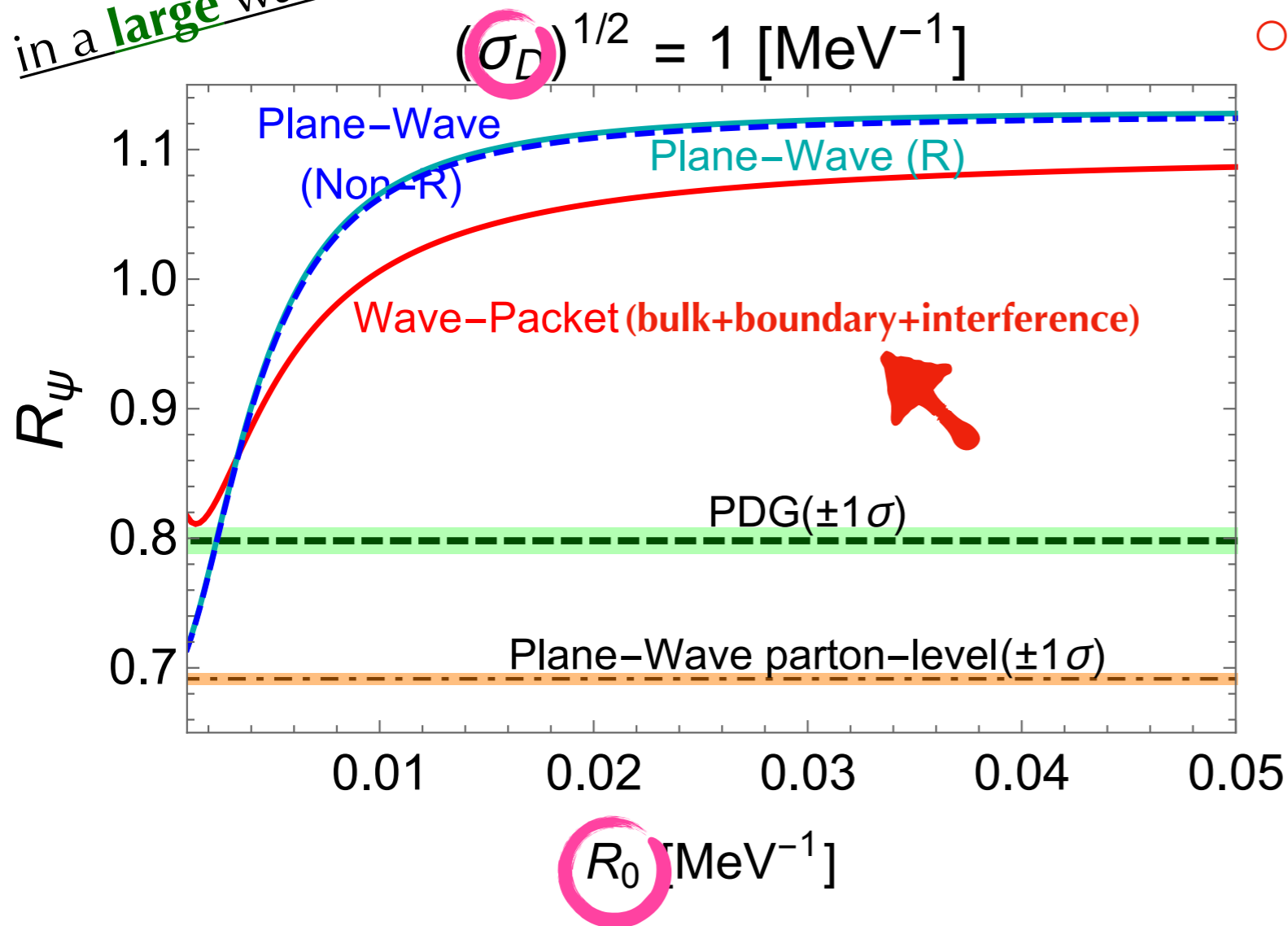
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in a **large** wave packet

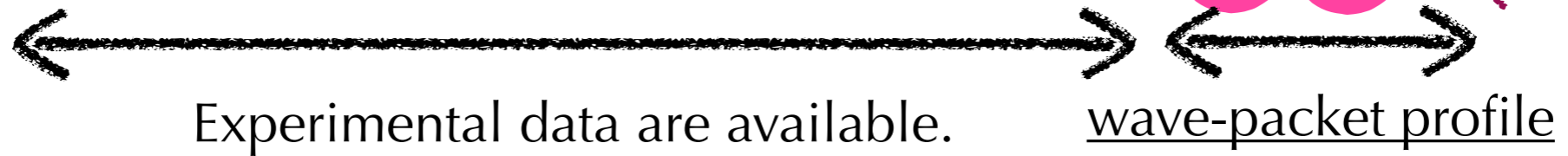


○ Bulk-only wave-packet result is far away from PDG (outside the shown region), also does not make sense.
⇒ Boundary part should be taken.

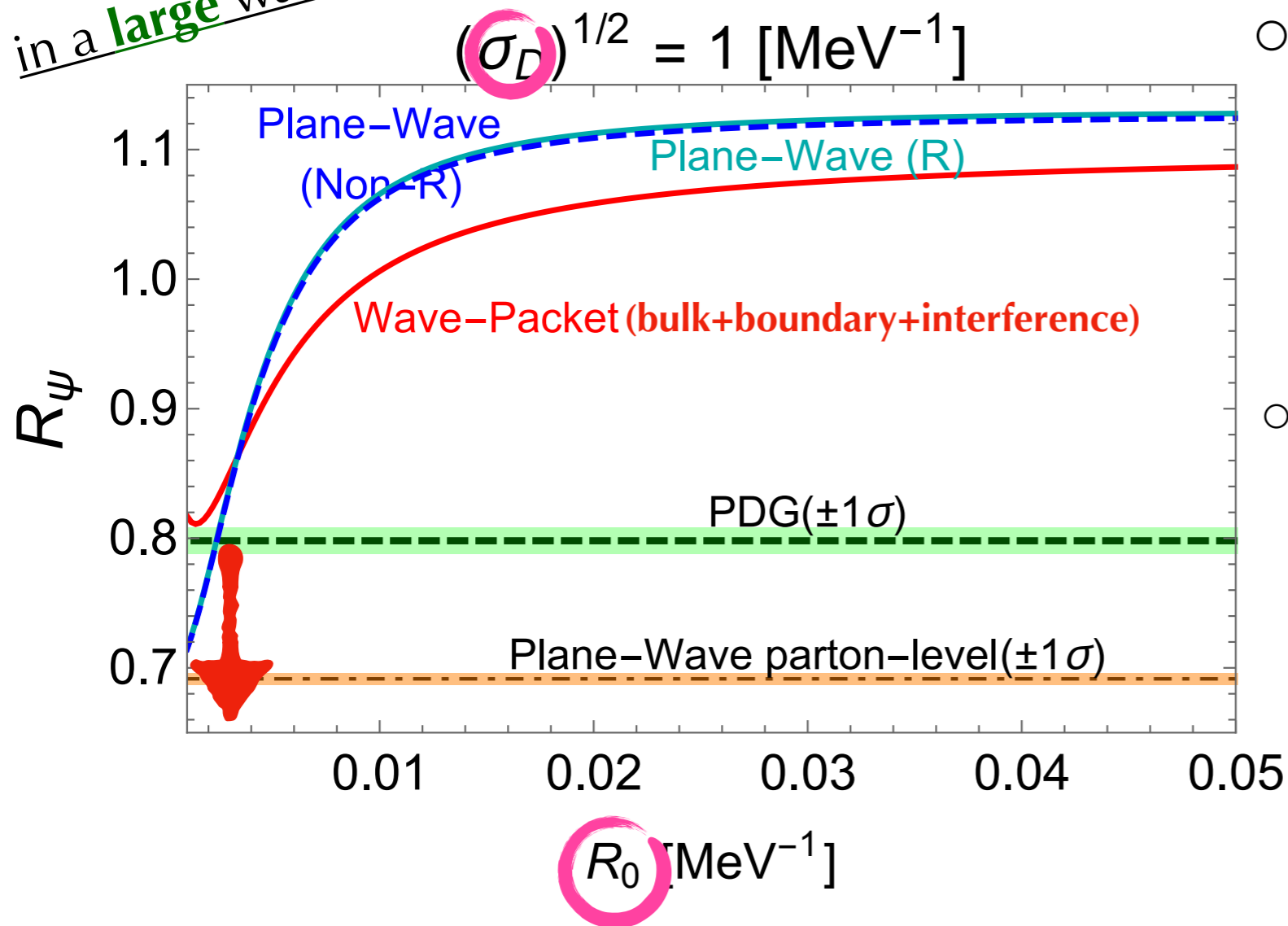
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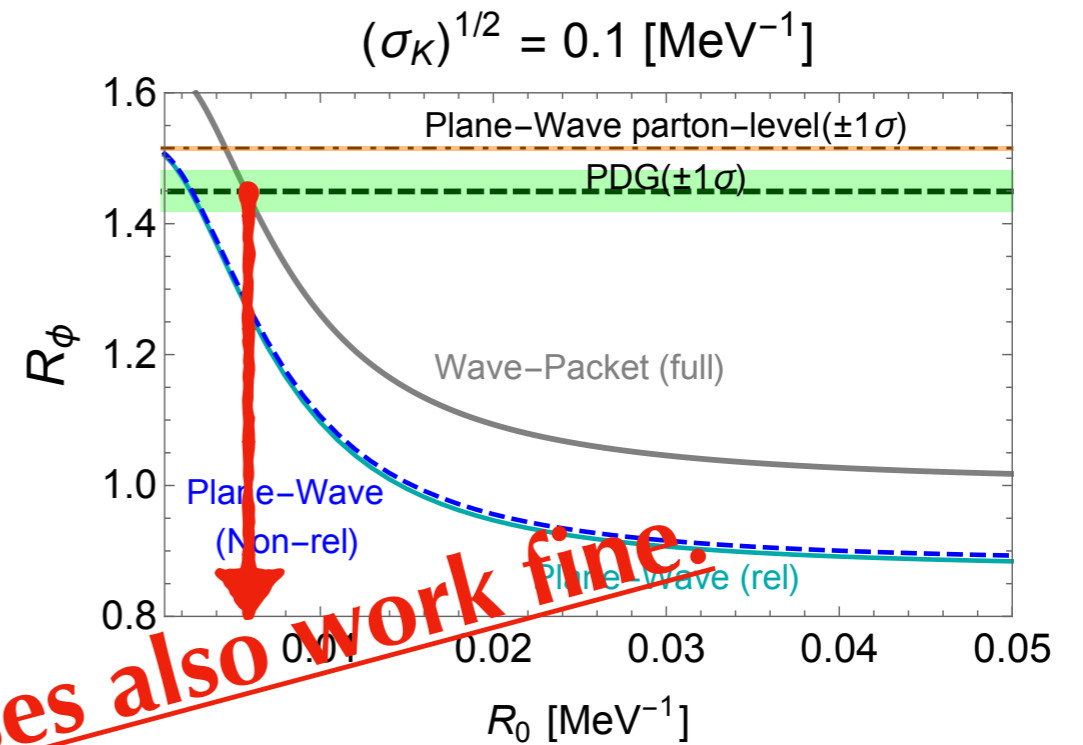
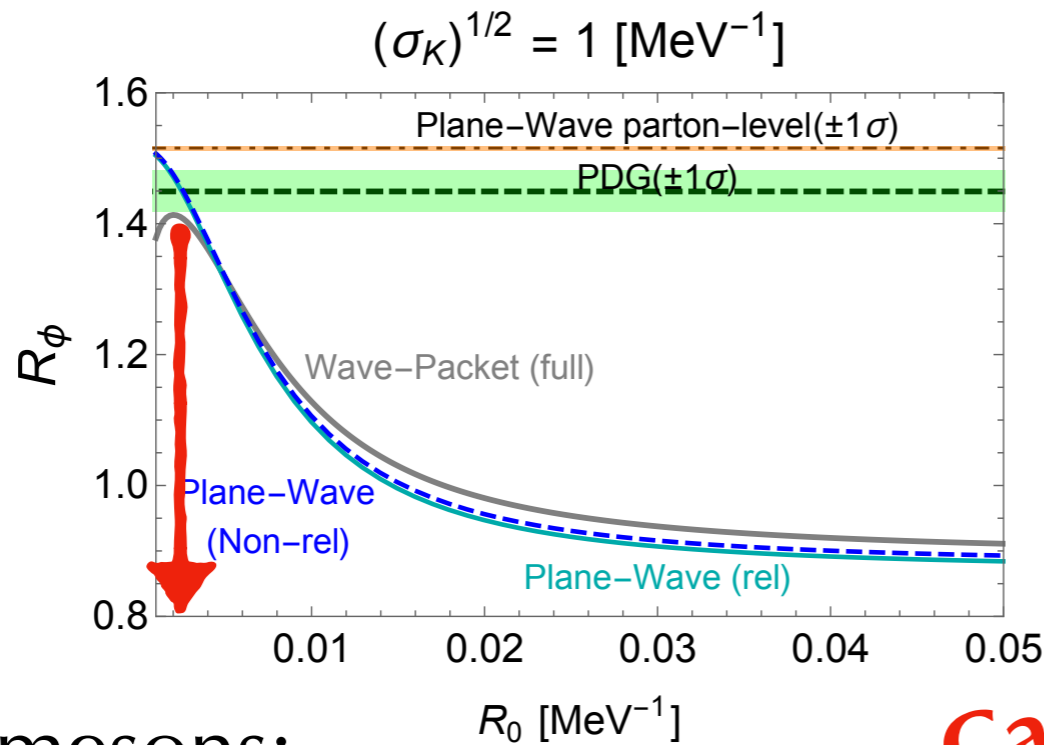
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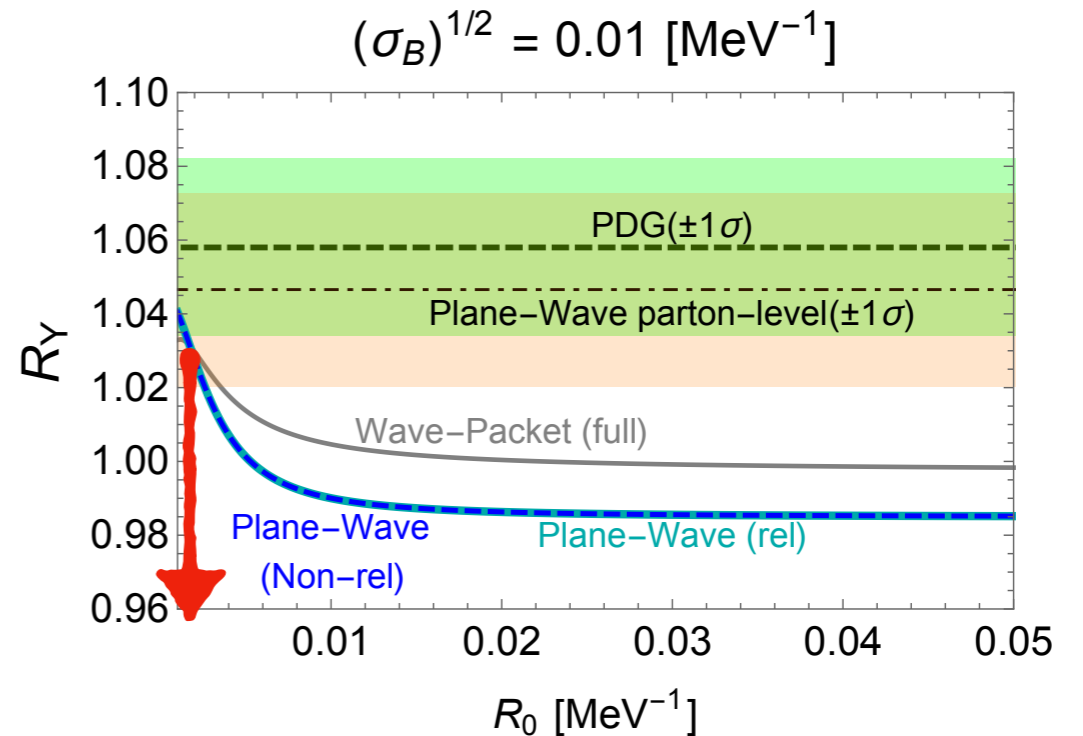
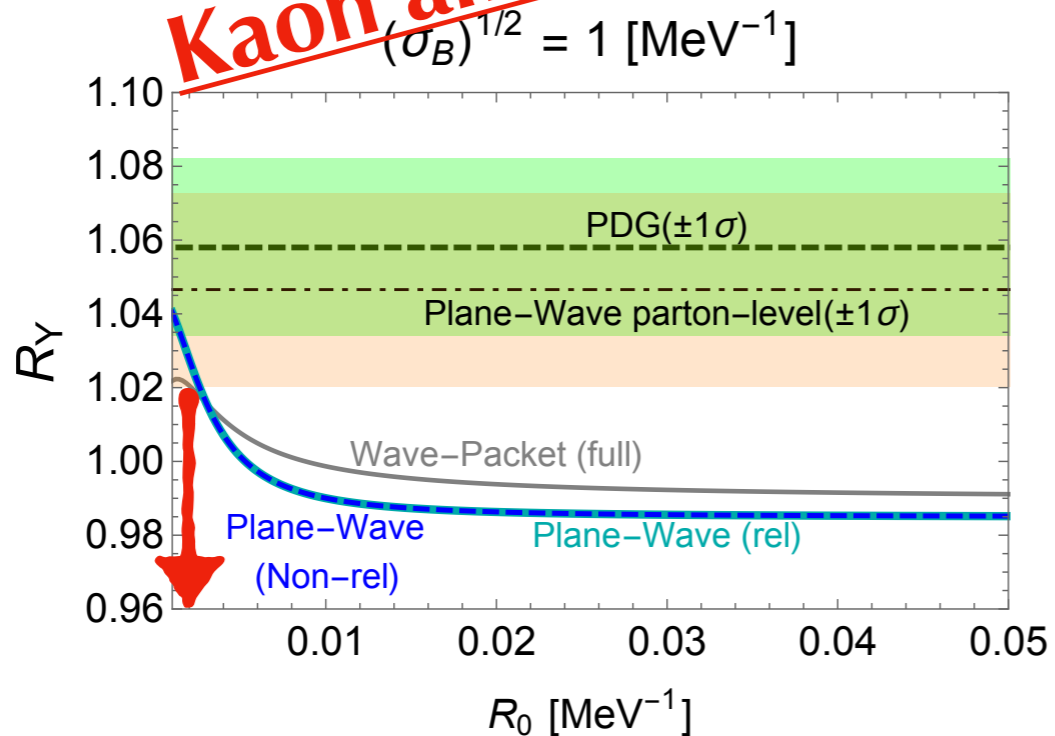
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- **R_0 for explaining PDG is similar for smaller and bigger wave packets.**

Predictions for the Ratio

□ For Kaons:



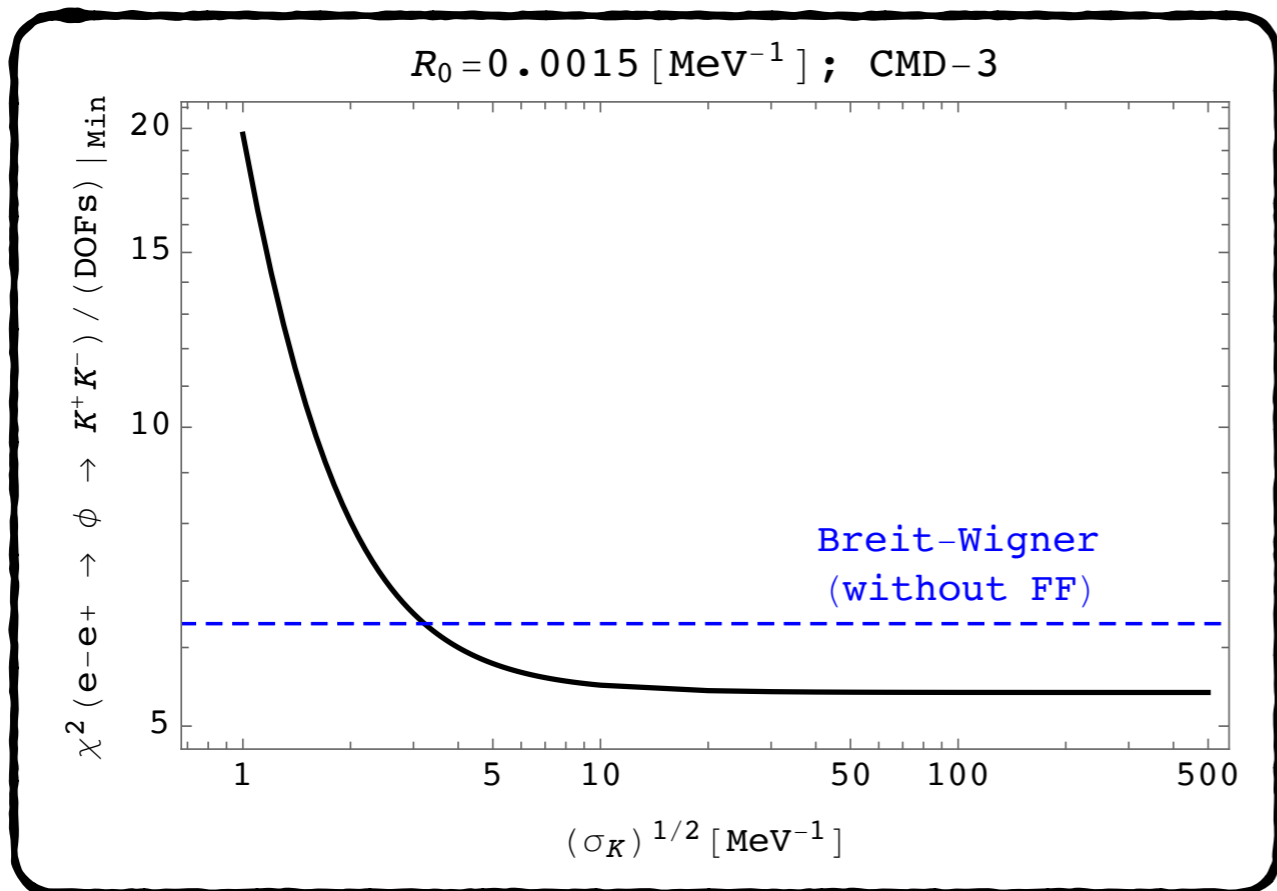
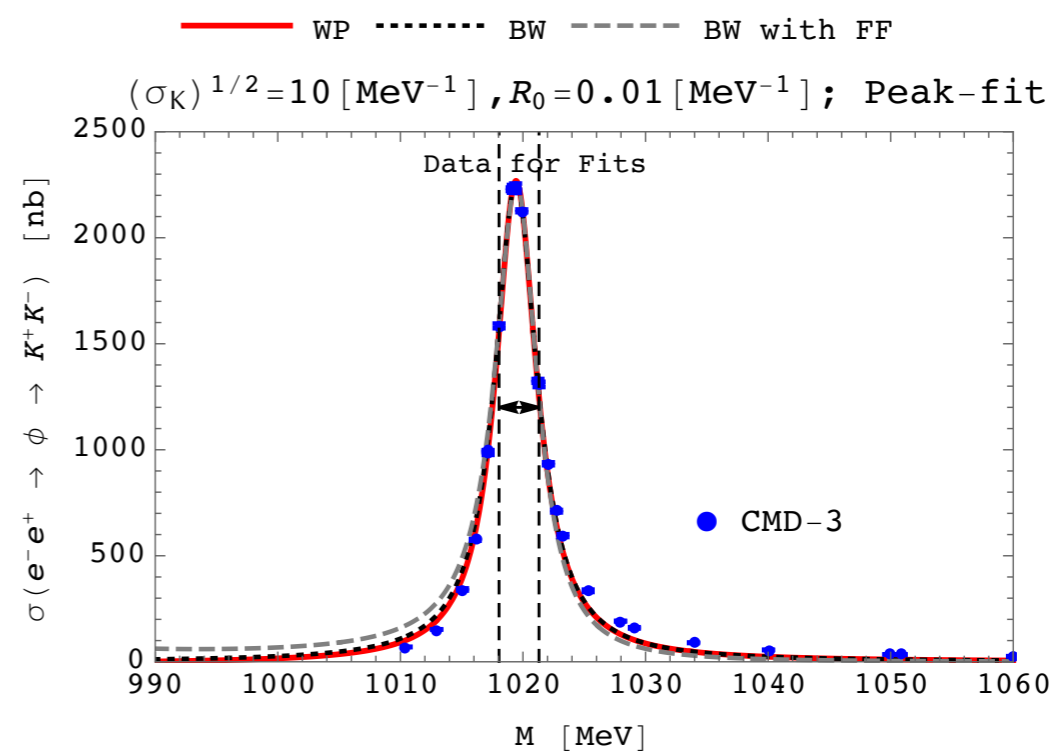
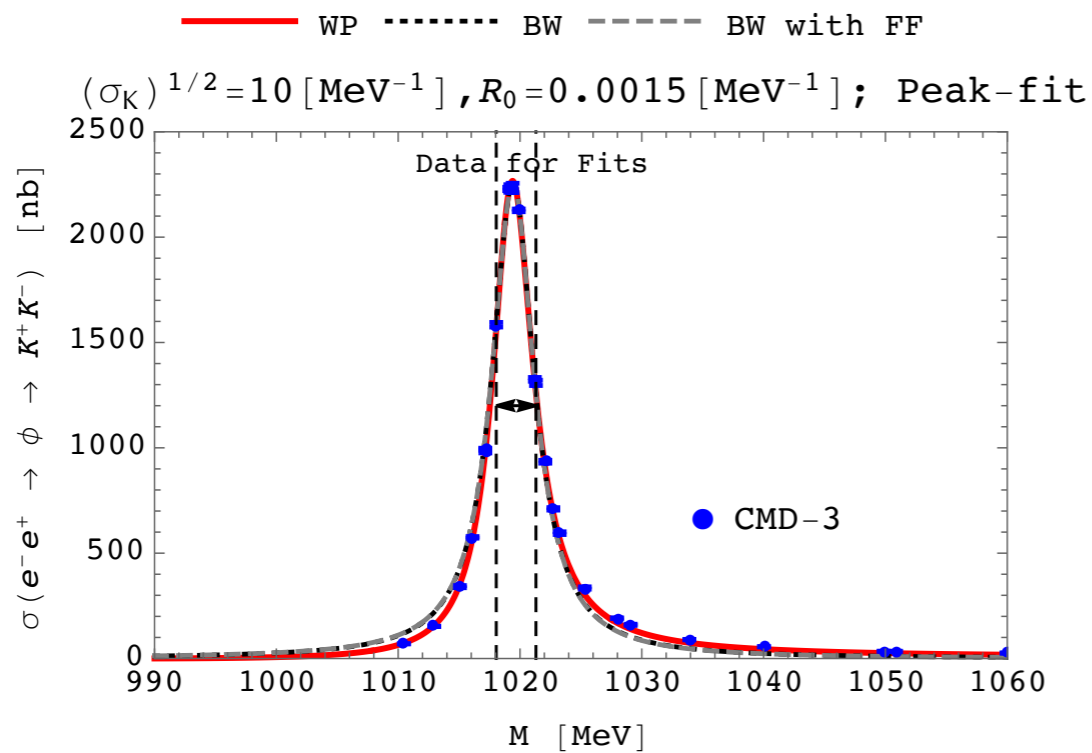
□ For B-mesons:



Kaon and B-meson Cases also work fine.

Constraint via Resonant shape

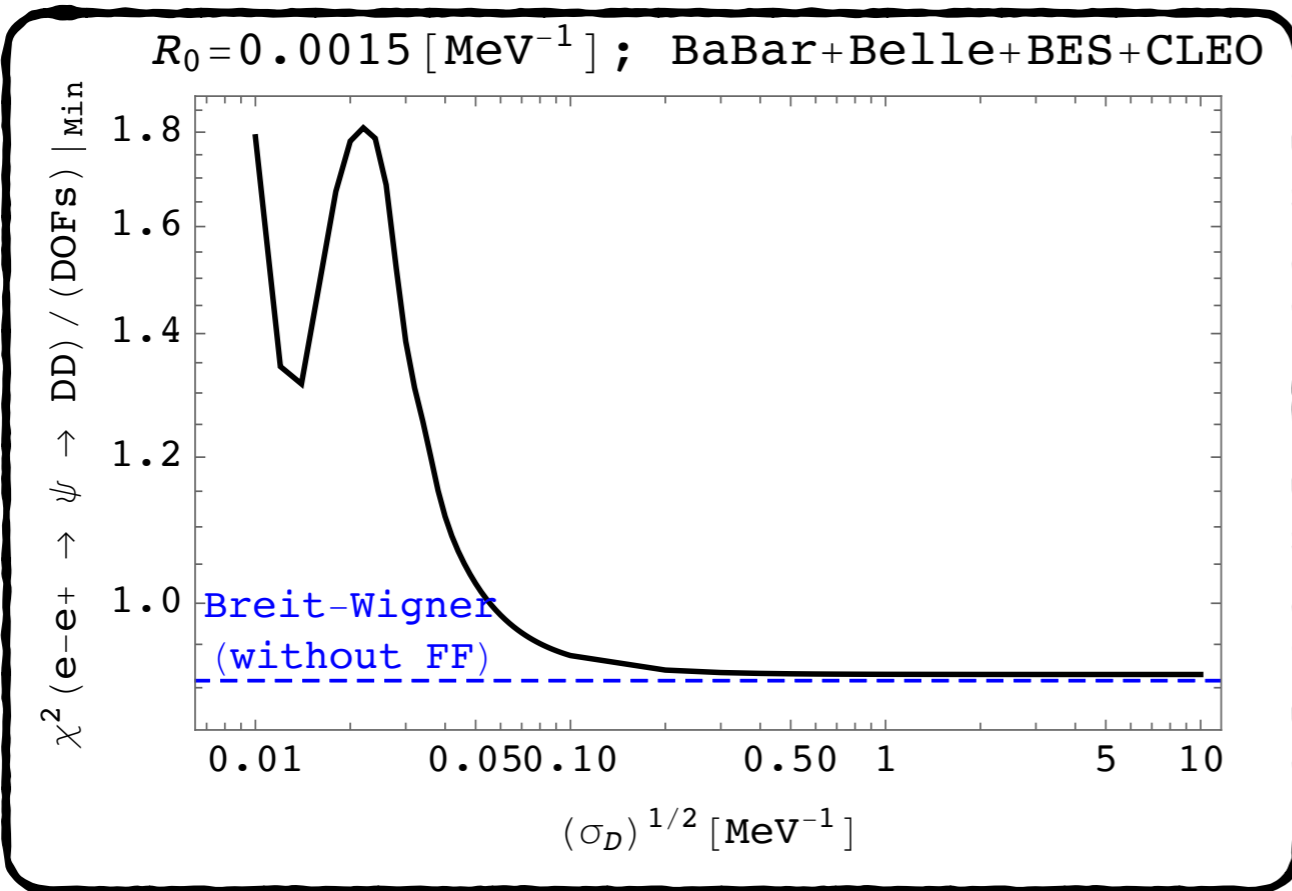
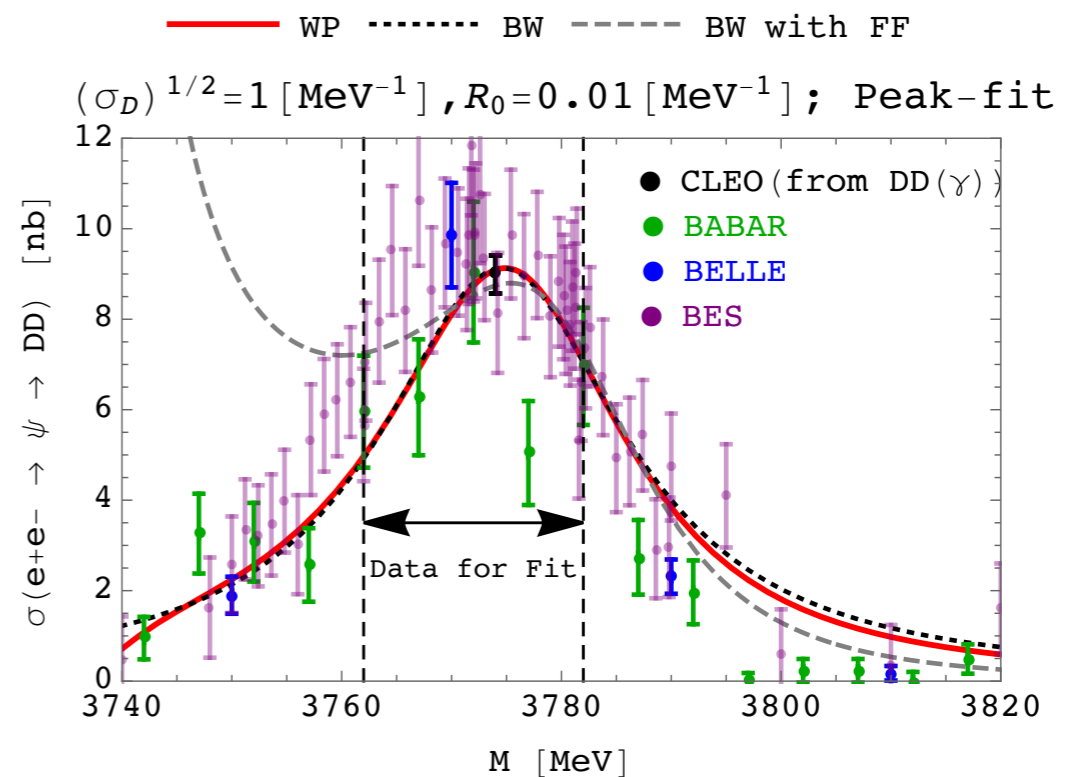
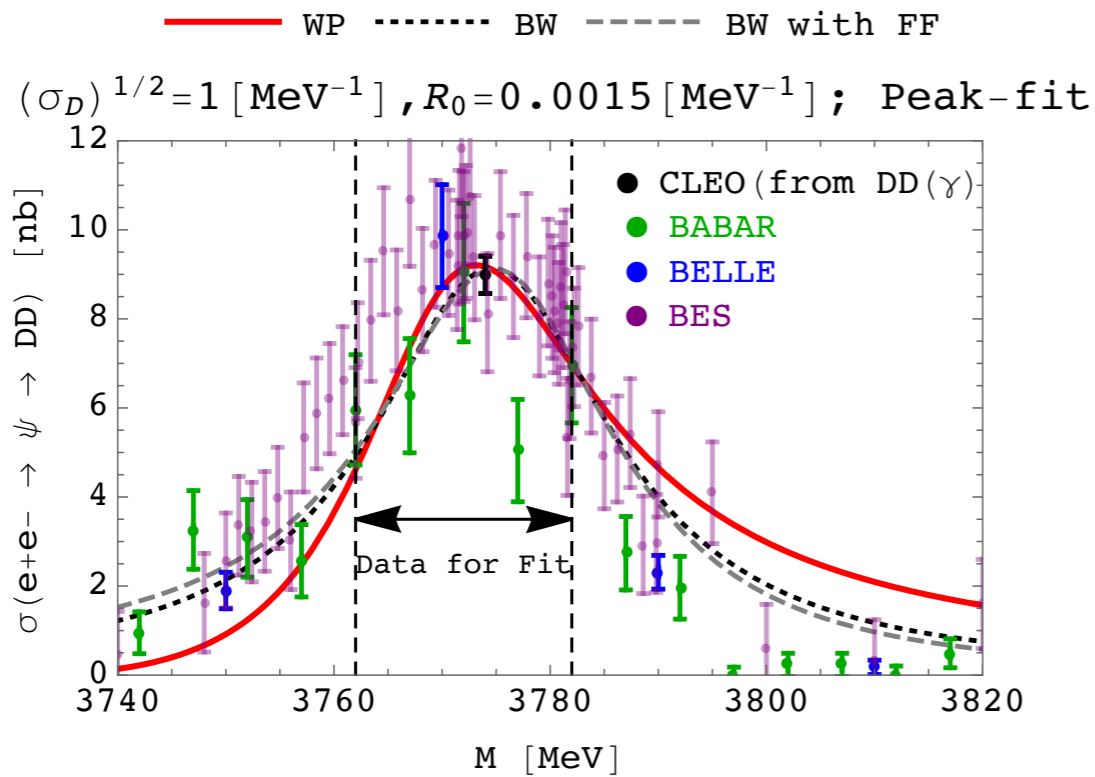
□ via $e^-e^+ \rightarrow \phi \rightarrow K^+K^-$



Note: $\sigma(e^-e^+ \rightarrow \phi)$ and m_ϕ are determined by statistical fits.

Constraint via Resonant shape

□ via $e^-e^+ \rightarrow \psi \rightarrow 2D (D^+D^- \text{ and } D^0\bar{D}^0)$

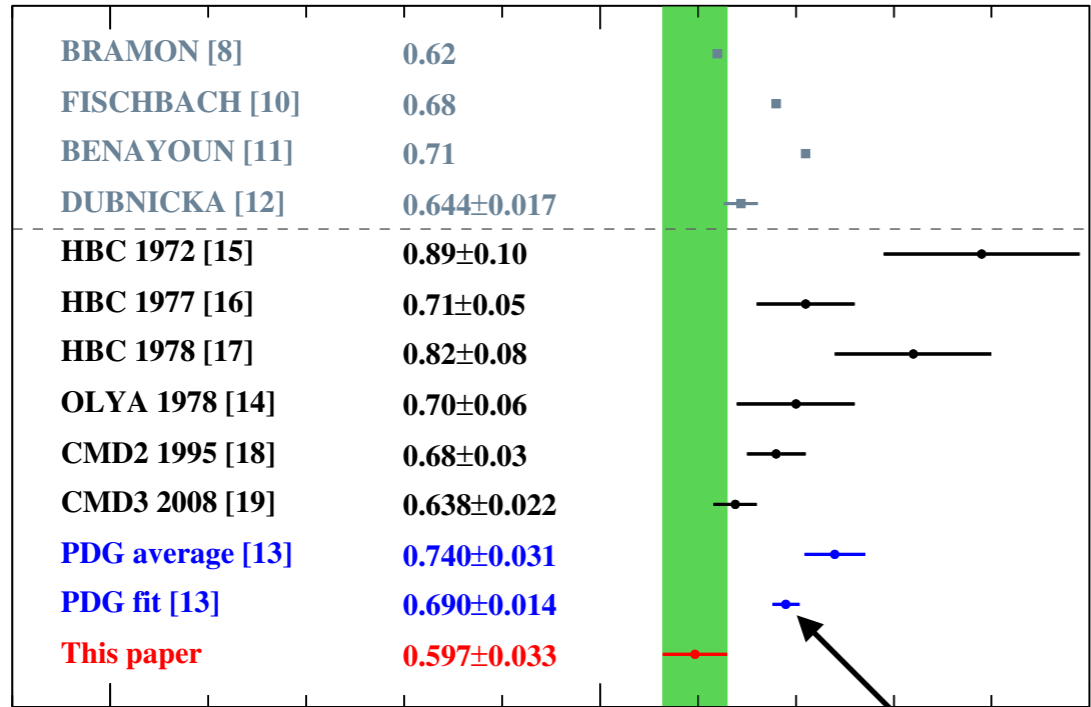


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Brief Note on Recent Updates

[BESIII, hep-ex/2503.11383]

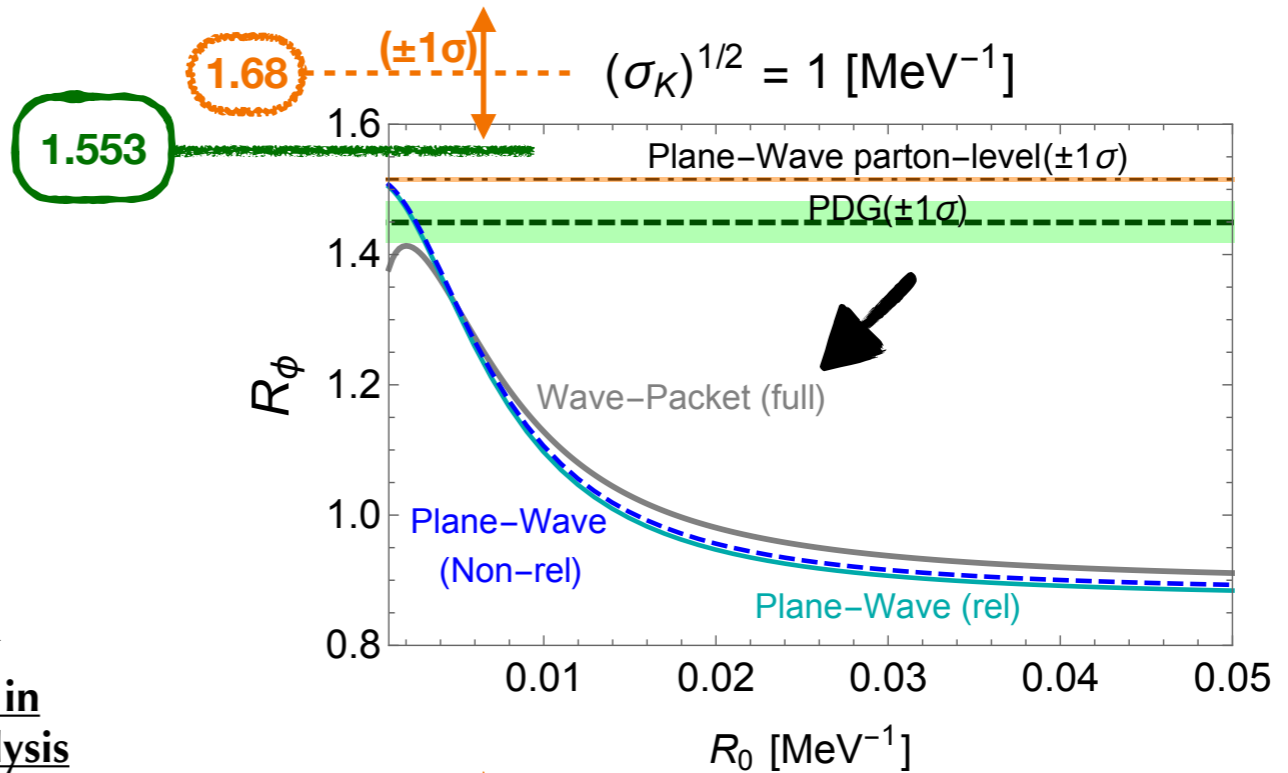
Recent BESIII experimental update:



$$(R_\phi)^{-1} = \frac{\text{Br}(\phi \rightarrow K_S^0 K_L^0)}{\text{Br}(\phi \rightarrow K^+ K^-)}$$

Target in our analysis

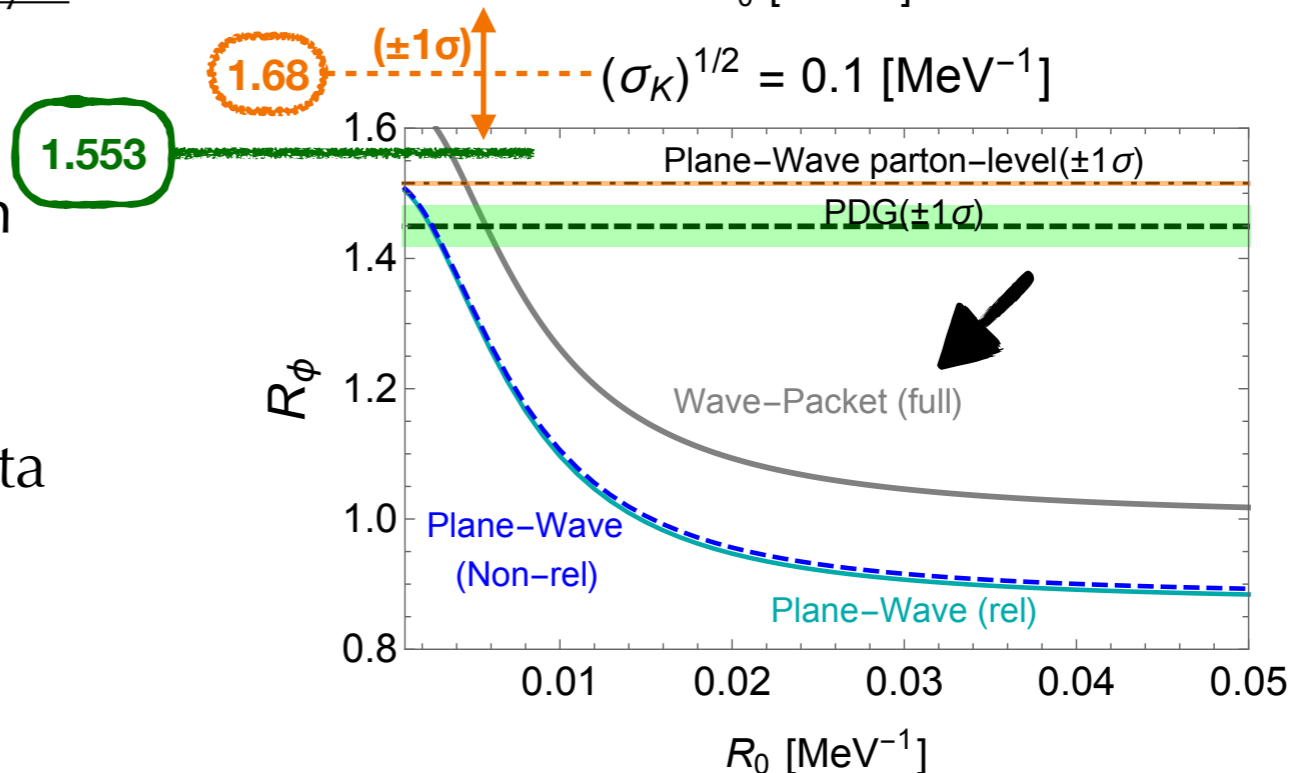
$$\Rightarrow (R_\phi) = \frac{\text{Br}(\phi \rightarrow K^+ K^-)}{\text{Br}(\phi \rightarrow K_S^0 K_L^0)} = 1.68 \pm 0.09$$



[Phys. Rev. D110 (2024) 054019]

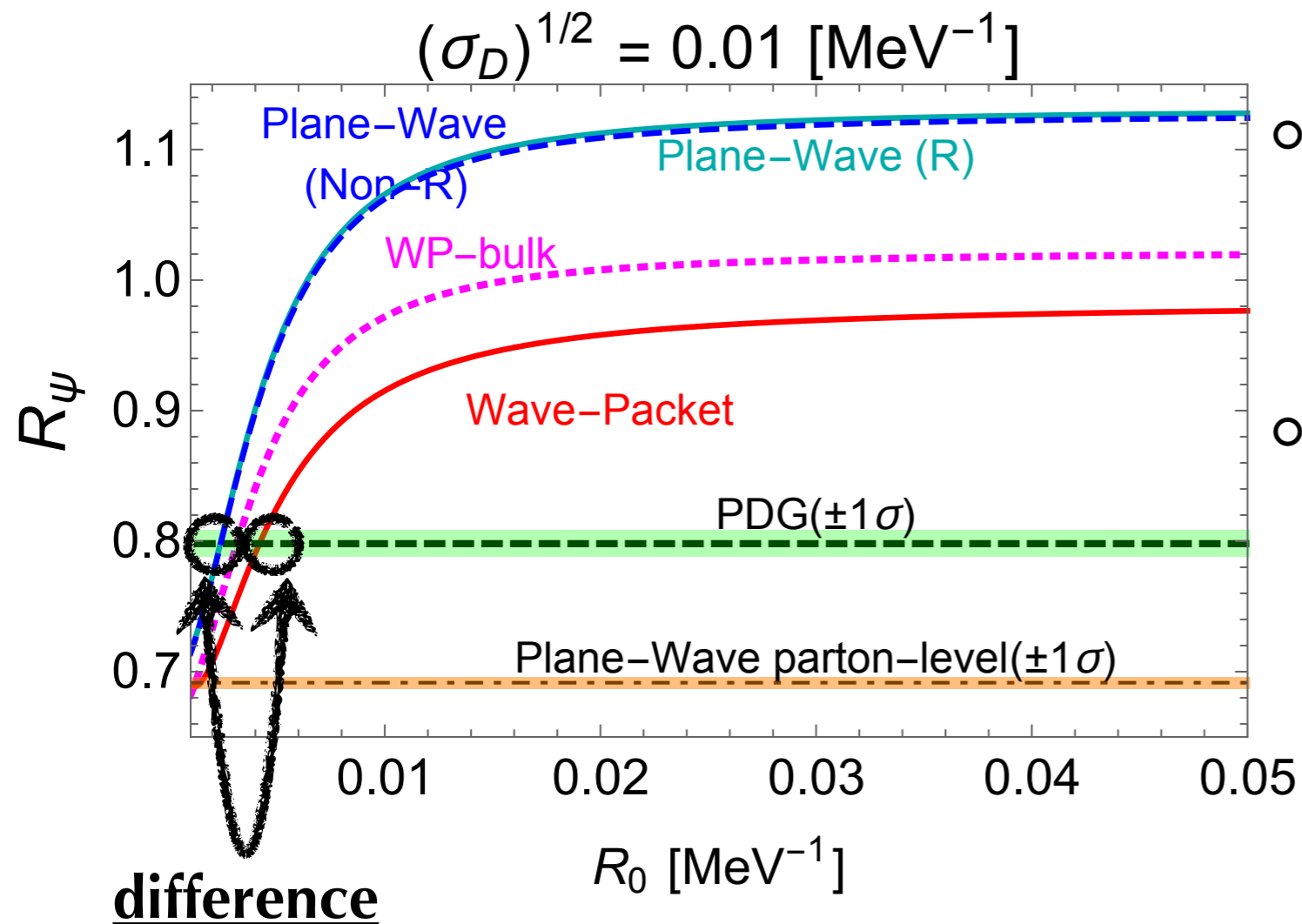
Recent extraction from cross-section measurements of $e^-e^+ \rightarrow K^+K^-$ and $e^-e^+ \rightarrow K_S^0 K_L^0$: $R_\phi = 1.553 \pm 0.040$

(Note: CMD-3, BABAR and BESIII data are used.)



Summary & Discussion

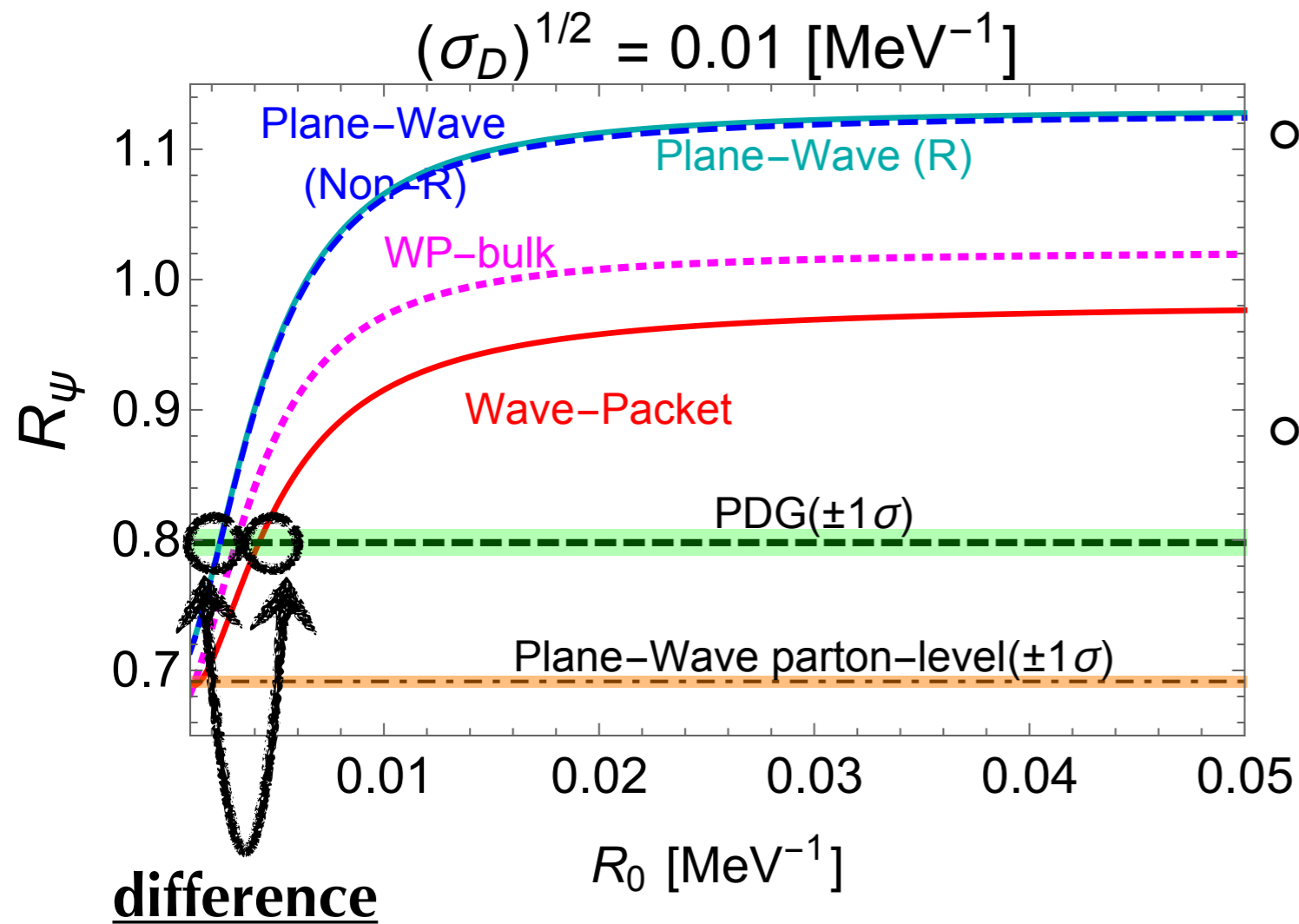
1. The S-matrix in Gaussian wave packet contains **full information** of the **quantum particles**. → **More informative & regularised**.
2. Characterising S-matrix, in particular, “**bulk**” and “**boundary**”.
- 3.



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- Can we **distinguish** the “wave-packet” correction from the plane-wave part (in this variable or others)?
→ Further discussion is necessary.

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THANK YOU!

BACKUP SLIDES

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DETAILS

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$$\hat{\mathcal{H}}_{\text{int,eff}}^{(I)} = ig_{V^+} \mathcal{V}^\mu [\mathcal{P}^+ \partial_\mu \mathcal{P}^- - \mathcal{P}^- \partial_\mu \mathcal{P}^+] \\
 + ig_{V^0} \mathcal{V}^\mu [\mathcal{P}^0 \partial_\mu \bar{\mathcal{P}}^0 - \bar{\mathcal{P}}^0 \partial_\mu \mathcal{P}^0]$$

$$|g_{\text{eff}}|^2 := \frac{g_V^2}{3} \sum_{\lambda_0} |\varepsilon_\mu(P_0, \lambda_0) (P_1^\mu - P_2^\mu)|^2 = \frac{g_V^2}{3} (\mathbf{P}_1 - \mathbf{P}_2)^2.$$

$g_+ = g_0$ ($\rightarrow g$) is suggested via the isospin symmetry ($u \leftrightarrow d$)

Form of Gaussian wave-packet S-matrix

(SKIPPABLE)
DETAILS

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$$\times e^{-\frac{\Gamma_V}{2}(\mathcal{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2\sigma_t}{8}} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)$$

normalisation factors
of free Gaussians

overlaps of the wave packets
(including approximated
Energy-Momentum conservation)

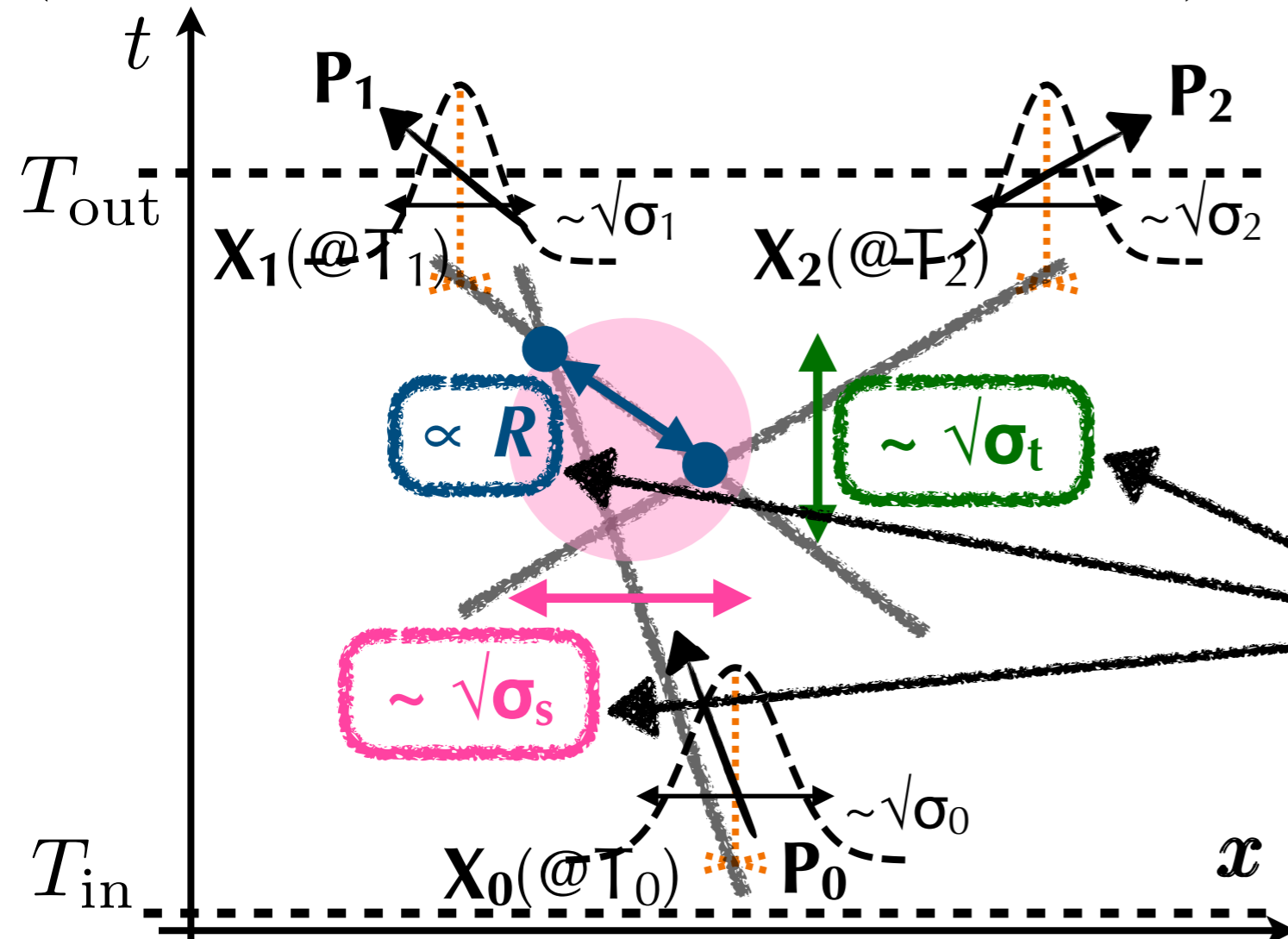
(SKIPPABLE) Form of Gaussian wave-packet S-matrix

DETAILS

$$\checkmark S_{V \rightarrow P\bar{P}} = ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi\sigma_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta P)^2 - \frac{\mathcal{R}}{2} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathcal{T})} \\
 \times e^{-\frac{\Gamma_V}{2}(\mathcal{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2\sigma_t}{8} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)}$$

• Geometrical variables characterise S.

$$(\delta\omega \sim \delta E := E_{\text{out}} - E_{\text{in}}, \delta P := P_{\text{out}} - P_{\text{in}})$$



They are functions of
 X_i, P_i, σ_i ($i=0,1,2$)

(SKIPPABLE) Form of Gaussian wave-packet S-matrix

DETAILS

$$\checkmark S_{V \rightarrow P\bar{P}} = ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi\sigma_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta P)^2} \frac{\mathcal{R}}{2} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathcal{T}) \\
 \times e^{-\frac{\Gamma_V}{2}(\mathcal{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2\sigma_t}{8}} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)$$

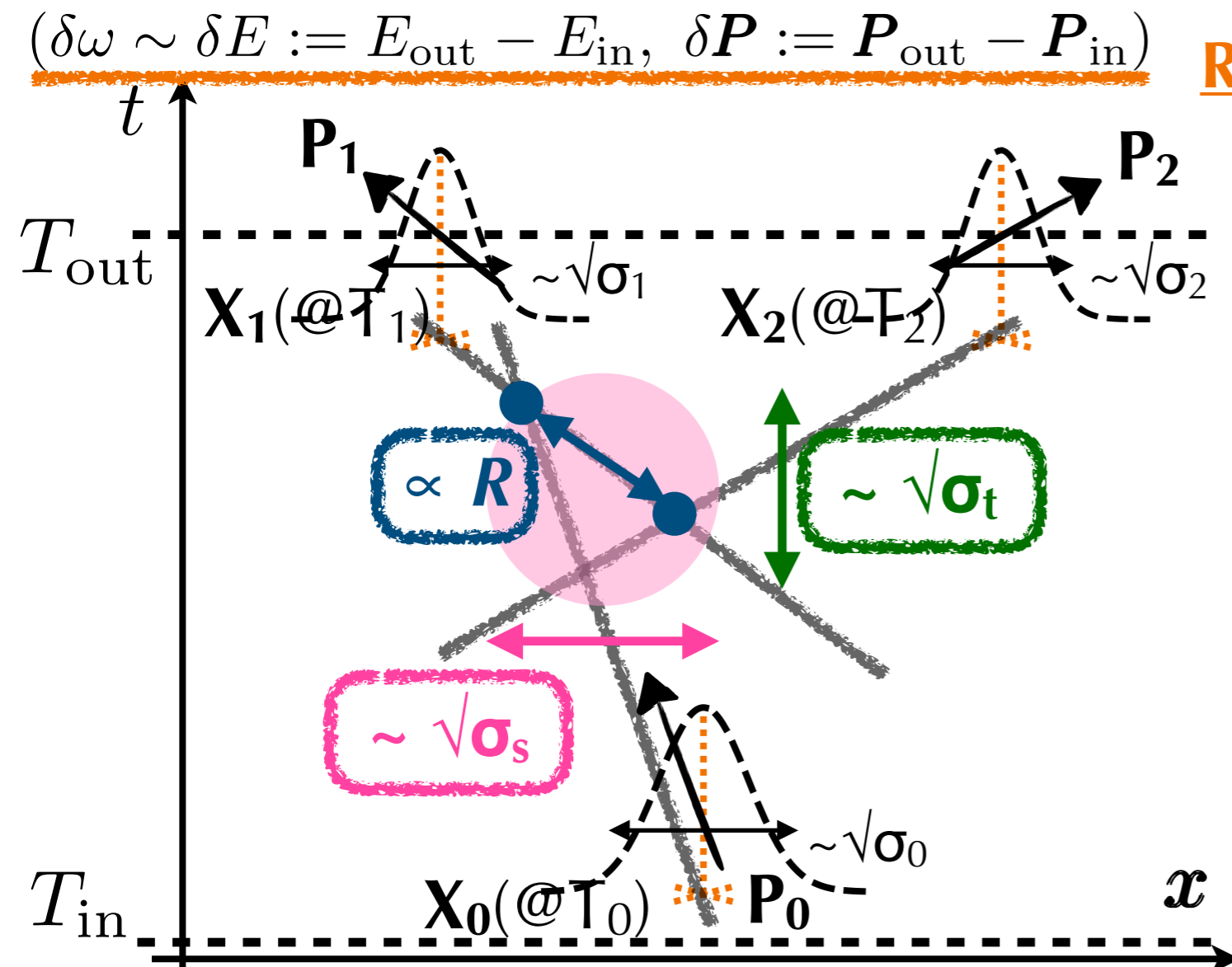
• Geometrical variables characterise S.

The limit ($\sigma_s \rightarrow \infty$ and $\sigma_t \rightarrow \infty$) \Rightarrow

Recovery of the energy-momentum conservation

Note:

$$\left(\sqrt{\frac{\sigma}{2\pi}} e^{-\frac{\sigma}{2}(p-p_0)^2} \xrightarrow{\sigma \rightarrow \infty} \delta(p - p_0) \right)$$



(SKIPPABLE) Form of Gaussian wave-packet S-matrix

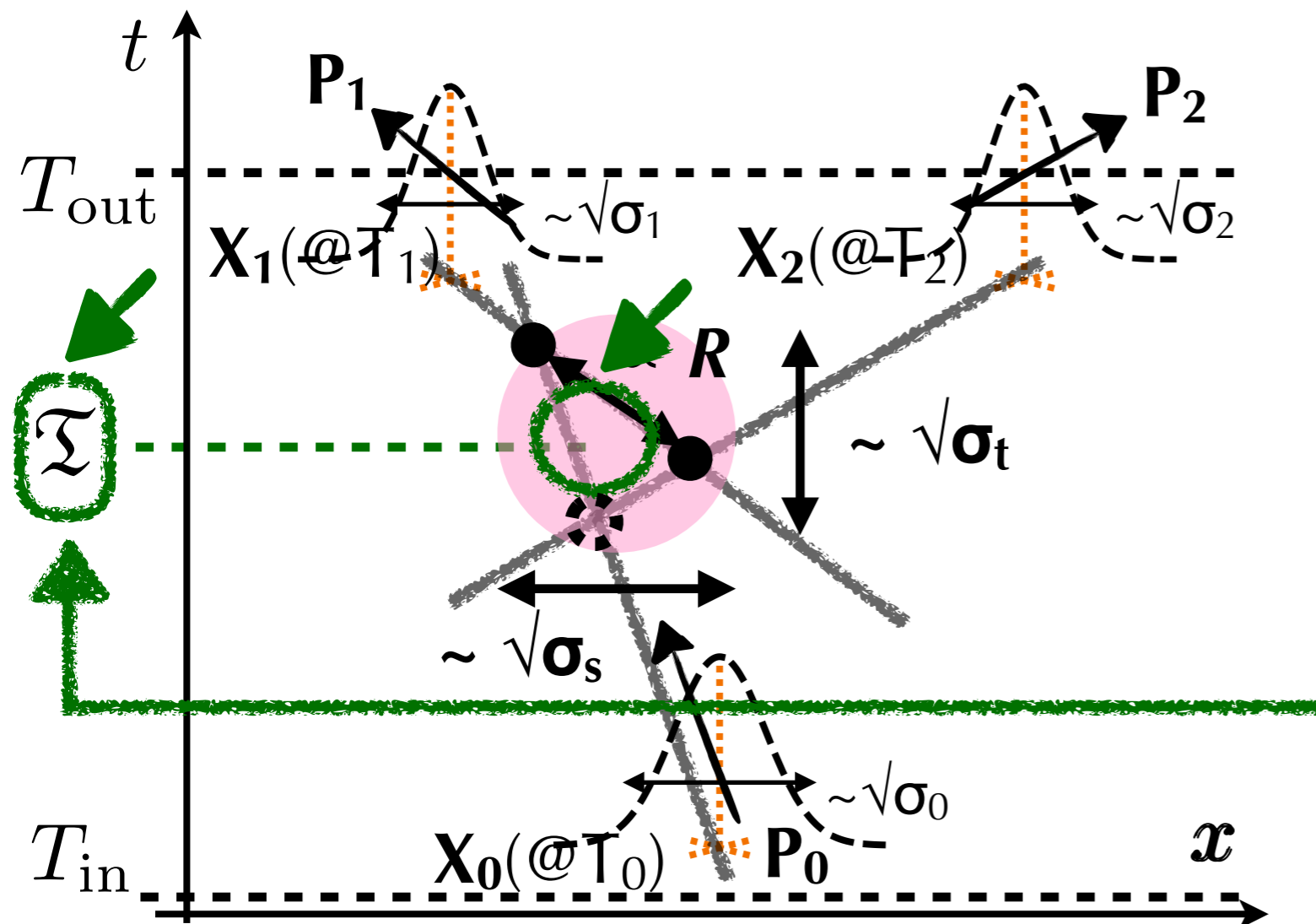
DETAILS

$$\checkmark S_{V \rightarrow P\bar{P}} = ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi\sigma_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta\mathbf{P})^2 - \frac{\mathcal{R}}{2}} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathcal{T})$$

$$\times e^{-\frac{\Gamma_V}{2}(\mathcal{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2\sigma_t}{8}} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)$$

\mathcal{T} : time of overlap (around which three wave packets overlap).

“window function”



$$\mathcal{T} := \sigma_t \frac{\overline{\mathbf{V}} \cdot \overline{\mathbf{x}} - \overline{\mathbf{V}} \cdot \mathbf{x}}{\sigma_s}$$

$$\mathbf{x}_A := \Xi_A(0) (= \mathbf{X}_A - \mathbf{V}_A X_A^0)$$

determined by the trajectories
(configurations of
external particles)

(SKIPPABLE)
DETAILS

Form of Gaussian wave-packet S-matrix

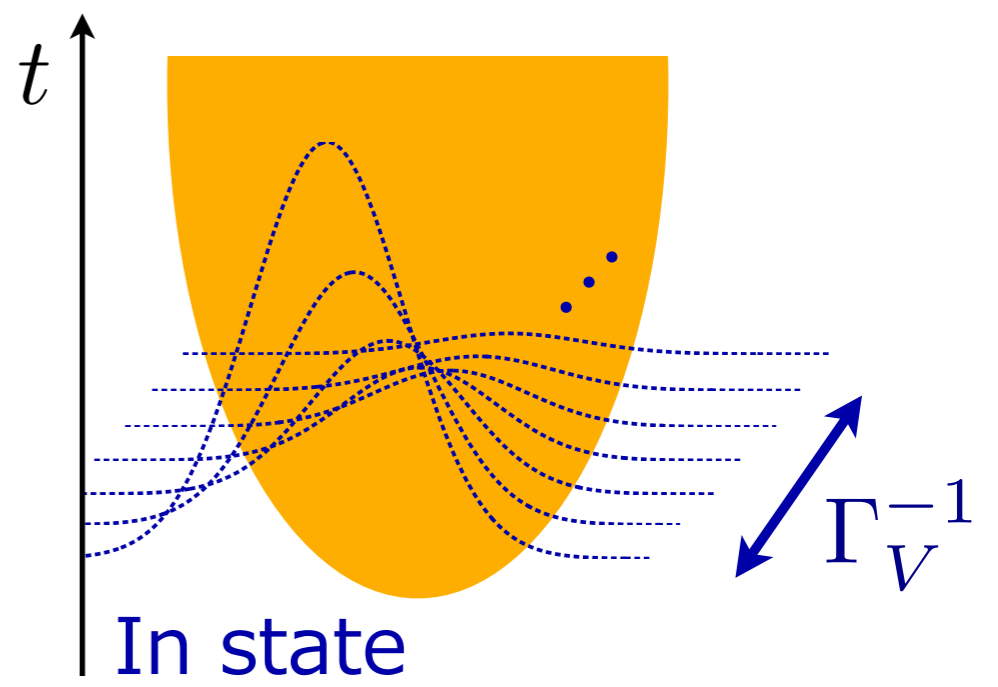
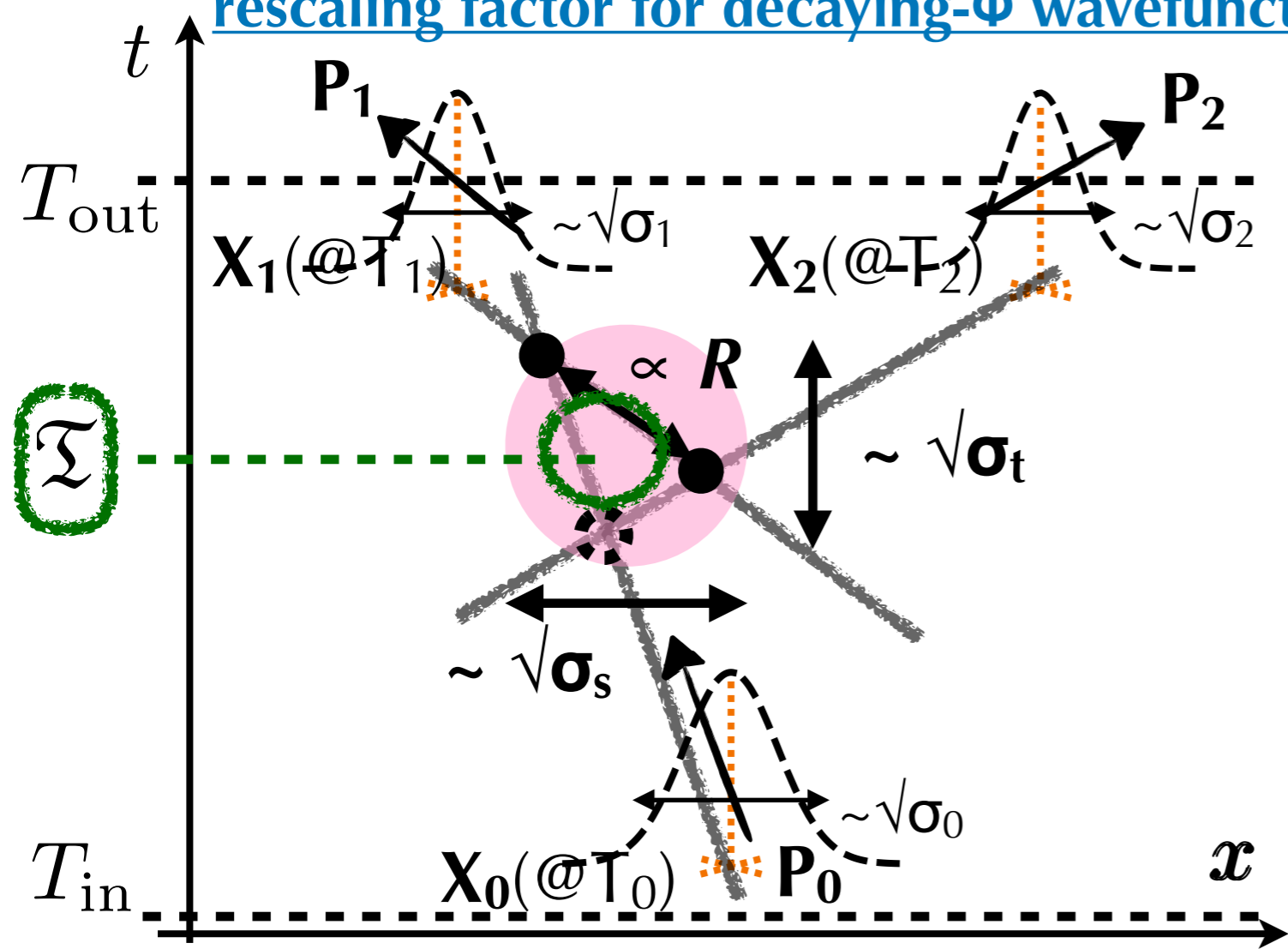
$$\checkmark S_{V \rightarrow P\bar{P}} = ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi E_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta\mathbf{P})^2 - \frac{\mathcal{R}}{2}} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathcal{T}) \\
 \times e^{-\frac{\Gamma_V}{\sigma_s}(\mathcal{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2}{\sigma_s} \mathcal{T}} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)$$

\mathcal{T} : time of overlap (around which three wave packets overlap).

“window function”

rescaling factor for decaying- Φ wavefunction

The off-shell-ness (decaying nature) of the initial state is taken into account (Weisskopf-Wigner Approx.).



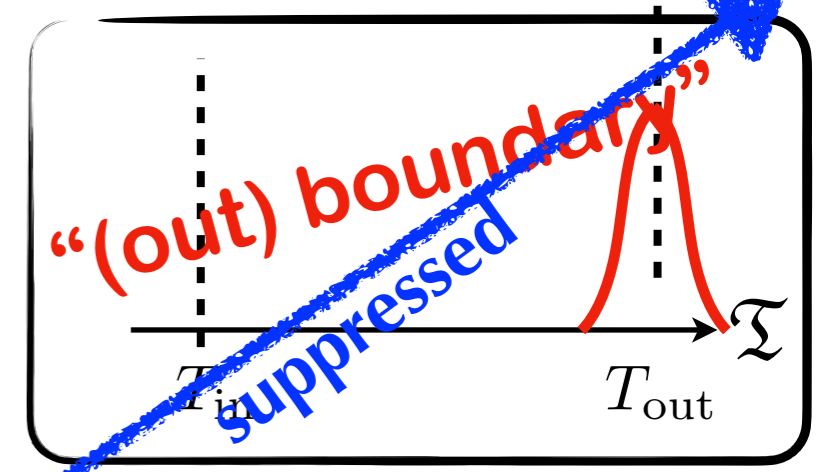
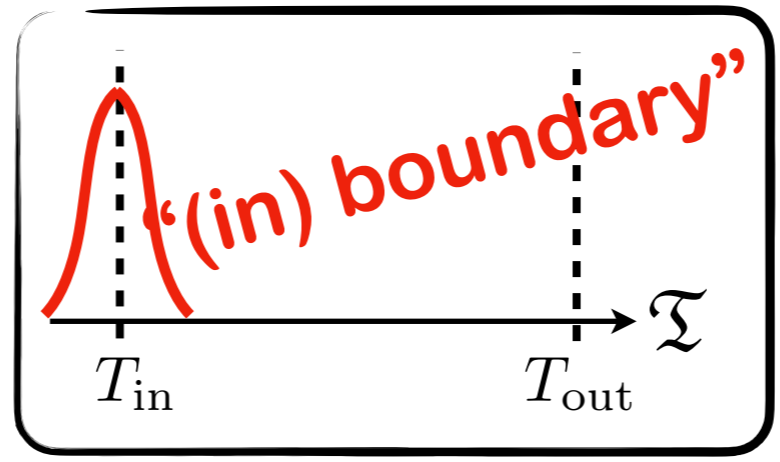
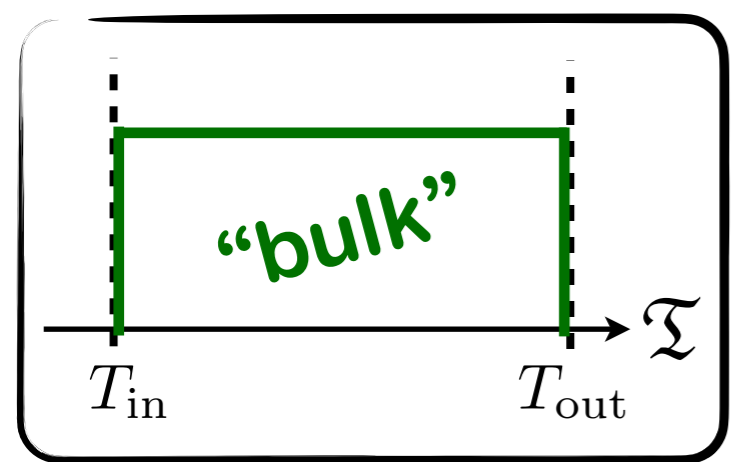
(SKIPPABLE) Form of Gaussian wave-packet S-matrix

DETAILS

$$\checkmark S_{V \rightarrow P\bar{P}} = ig_{\text{eff}} N_V \left(\prod_{A=0}^2 \frac{1}{\sqrt{2E_A}} \left(\frac{1}{\pi\sigma_A} \right)^{3/4} \right) e^{-\frac{\sigma_t}{2}(\delta\omega)^2 - \frac{\sigma_s}{2}(\delta P)^2 - \frac{\mathcal{R}}{2}} (2\pi\sigma_s)^{3/2} \sqrt{2\pi\sigma_t} G(\mathfrak{T}) \\
 \times e^{-\frac{\Gamma_V}{2}(\mathfrak{T} - T_0 + i\sigma_t\delta\omega) + \frac{\Gamma_V^2\sigma_t}{8}} \tilde{F}(|\mathbf{V}_1 - \mathbf{V}_2|)$$

\mathfrak{T} : time of overlap (around which three wave packets overlap).

“window function”



(by taking $T_{\text{out}} \rightarrow \infty$)

$$G(\mathfrak{T}) \simeq W(\mathfrak{T}) - \frac{1}{2} e^{-\frac{(\mathfrak{T} - T_{\text{in}} - \frac{\Gamma_V\sigma_t}{2})^2}{2\sigma_t} + \frac{\sigma_t}{2}(\delta\omega)^2 - i\delta\omega(\mathfrak{T} - T_{\text{in}} - \frac{\Gamma_V\sigma_t}{2})} \sqrt{\frac{2\sigma_t}{\pi}} \frac{1}{\mathfrak{T} - T_{\text{in}} - \frac{\Gamma_V\sigma_t}{2} + i\sigma_t\delta\omega} \\
 + \frac{1}{2} e^{-\frac{(\mathfrak{T} - \frac{\Gamma_V\sigma_t}{2} - T_{\text{out}})^2}{2\sigma_t} + \frac{\sigma_t}{2}(\delta\omega)^2 - i\delta\omega(\mathfrak{T} - \frac{\Gamma_V\sigma_t}{2} - T_{\text{out}})} \sqrt{\frac{2\sigma_t}{\pi}} \frac{1}{\mathfrak{T} - T_{\text{out}} - \frac{\Gamma_V\sigma_t}{2} + i\sigma_t\delta\omega}$$